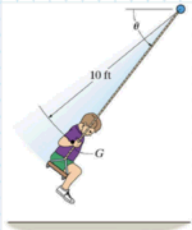
-001 P1 Grading (34 Points)

Signs on magnitude values do not have to match, but vector arrows must be in same direction.

xx/34

When $\theta = 30°$, the boy's center of mass G is momentarily at rest. Using force and acceleration methods, determine (a) his velocity and (b) his acceleration when $\theta = 70°$, writing these in vector form using normal and tangential components. Also, find (c) the tension in each of the two supporting cords of the swing at this position. The boy has a weight of 50 lb. Neglect his size and the mass of the seat and cords.



FIND (a) v, (b) a, (c) T @ $\theta = 70°$

KNOWN $v = 0$ @ $\theta = 30°$

$W = 50$ lbf

2 pt for writing **FIND** and listing all variables to find

$W = mg \rightarrow m = \frac{W}{g}$

$m = \frac{50}{32.2} = 1.553 \ \frac{lbf \cdot s^2}{ft}$

SOLUTION

① $+\nearrow \Sigma F_n = ma_n \rightarrow 2T - 50\sin\theta = 1.553 a_n$

$a_n = f(\theta) \neq const$ 2 pt for formula

② $\overset{+}{\searrow} \Sigma F_t = ma_t \rightarrow 50\cos\theta = 1.553 a_t$

$a_t = f(\theta) \neq const$ 2 pt for formula

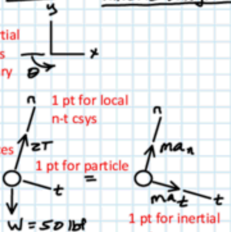
FBD **Kinetic diagram**

1 pt for inertial csys, theta is not necessary

1 pt for local n-t csys

1 pt for forces

1 pt for particle



1 pt for inertial vector or components

Apply kinematics to ① & ②

$a_n = \frac{v^2}{\rho}$ 2 pt for formula

① $\rightarrow 2T - 50\sin\theta = 1.553\frac{v^2}{10}$

$\rightarrow 2T - 50\sin\theta = 0.1553 v^2$

$a_t = \frac{50}{1.553}\cos\theta = 32.196\cos\theta$

4 pt for miscellaneous work

$a_t ds = v dv \rightarrow \int_{s_1}^{s_2} a_t ds = \int_{v_1}^{v_2} v dv$

2 pt for formula

$s = \theta r \rightarrow ds = r d\theta$

③ $\int_{\theta_1}^{\theta_2} a_t r d\theta = \int_{v_1}^{v_2} v dv$

2 pt for change of variables: s to Θ or Θ to s.

$50\cos 70° = 17.10$ lbf

$50\sin 70° = 46.98$ lbf

② + ③ $\rightarrow \int_{30°}^{70°} (10) 32.196 \cos\theta \, d\theta = \int_0^{v_2} v dv$

① $\rightarrow T = \frac{1.553(28.31) + 50\sin 70°}{2}$

$\boxed{T = 45.5 \text{ lbf} \nearrow}$

1 pt for value
1 pt for units
1 pt for arrow
1 pt for box or oval

$321.96 \sin\theta \Big|_{30°}^{70°} = \frac{v^2}{2}\Big|_0^{v_2}$

$141.56 = 0.5 v_2^2 \rightarrow v_2 = 16.826 \frac{ft}{s} \rightarrow \boxed{v = 16.8 \hat{u}_t \frac{ft}{s}}$

1 pt for value
1 pt for vector notation
1 pt for units
1 pt for box or oval

$a_n = \frac{16.826^2}{10} = 28.3 \frac{ft}{s^2} \nearrow$, $a_t = 32.196 \cos 70° = 11.0 \frac{ft}{s^2} \searrow$

$\boxed{a = 11.0 \hat{u}_t + 28.3 \hat{u}_n \ \frac{ft}{s^2}}$

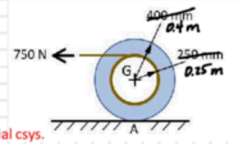
1 pt for t value
1 pt for n value
1 pt for vector notion
1 pt for units
1 pt for box or oval

-001 P2 Grading (34 Points)

Signs on magnitude values do not have to match, but vector arrows must be in same direction.

xx/34

A nonflexible rope is wrapped around the cylinder at the radius indicated and is pulled to the left with a force of 750 N. The cylinder has a mass of 80 kg and a radius of gyration of k_G = 0.2m. If the coefficients of static and kinetic friction at A are μ_s = 0.1 and μ_k = 0.08, respectively, determine (a) the angular acceleration of the cylinder, (b) the translational acceleration of its center of mass G, and (c) the friction force at A.



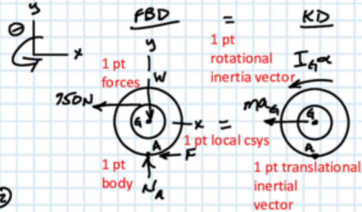
2 pt Heading FIND and list all 3 parameters to find

FIND (a) $\vec{\alpha}$, (b) $\vec{a}_G$, (c) F

KNOWN $m = 80\ kg$, $k_G = 0.2m$

$\mu_s = 0.1$, $\mu_k = 0.08$

1 pt inertial csys. Θ not req'd.



$W = mg = (80)(9.81) = 784.8\,N$

$I_G = mk^2 = 3.2\,kg\cdot m^2$

SOLUTION

$\overset{+}{\rightarrow} \Sigma F_x = ma_x \rightarrow -750 - F = -80\,a_G$ ①

2 pt

$+\uparrow \Sigma F_y = ma_y \rightarrow N_A - 784.8 = 0 \rightarrow N_A = 784.8N$ ②

2 pt

$+\circlearrowleft \Sigma M_G = I_G\alpha \rightarrow 0.25(750) - 0.4F = 3.2\alpha$

2 pt

$\rightarrow 0.4F + 3.2\alpha = 187.5$ ③

Unknowns: F, a_G, α

① & ③ → 2 eq, 3 unknowns

2 pt

1st Assume no slipping → $a_G = \alpha r_{G/A}$ & $F \le \mu_s N_A$

$a_G = 0.4\alpha$ ④

① → $-750 - F = -80(0.4\alpha) = -32\alpha \rightarrow F - 32\alpha = -750$ ⑤

⑤ & ③ → 2 eq & 2 unknowns: α, F

$F = -750 + 32\alpha$

$0.4(-750 + 32\alpha) + 3.2\alpha = 187.5$

$\rightarrow -300 + 12.8\alpha + 3.2\alpha = 187.5$

$\rightarrow 16\alpha = 487.5 \rightarrow \alpha = 30.47\ \frac{rad}{s^2} \circlearrowleft$

3 pt for miscellaneous work

④ → $a_G = 0.4(30.47) = 12.19\ \frac{m}{s^2} \leftarrow$

1 pt or

Check assumption of no slipping.

⑤ → $F = -750 + 32(30.47) = 225N$

$\mu_s N = 0.1(784.8) = 78.5N$ 1 pt

$\Big\}\ F \le \mu_s N$?

Assumption of no slipping is not valid. 1 pt

So, $F = \mu_k N$

1 pt value
1 pt units
1 pt direction
1 pt box

$= 0.08(784.8) = \boxed{62.78N \leftarrow}$

① → $750 + 62.78 = 80\,a_G$

$\rightarrow \boxed{a_G = 10.2\ \frac{m}{s^2} \leftarrow}$

③ → $0.4(62.78) - 187.5$

$= -3.2\alpha$

$\rightarrow \boxed{\alpha = 50.7\ \frac{rad}{s^2} \circlearrowleft}$

1 pt value
1 pt units
1 pt direction
1 pt box

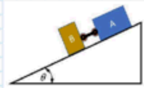
1 pt value
1 pt units
1 pt direction
1 pt box

-001 P3 Grading (34 Points)

4 pt for work or level of effort

$\frac{\ }{34}$

Masses A and B are connected by a link and have values of 12 kg and 6 kg, respectively. If they are placed on an inclined plane of angle θ = 45° and released from rest, determine (a) the force developed in the link and indicate tension or compression, (b) their acceleration, and (c) the time required for them to travel 0.5 m down the plane. The coefficients of kinetic friction between the blocks and the inclined plane are μ_A = 0.08 and μ_B = 0.28. Neglect the mass of the link and hinges.

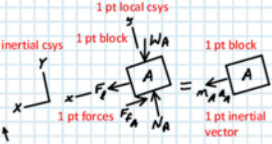


2 pt Heading FIND and list all 3 parameters to find

FIND (a) F_ℓ, (b) $\vec{a}$, (c) t @ $s_x = 0.5\,m$

KNOWN: $m_A = 12\,kg$, $m_B = 6\,kg$, $m_\ell = 0$

$\mu_A = 0.08$, $\mu_B = 0.28$



SOLUTION

@A $+\nwarrow \Sigma F_y = m_A a_{A_y} = 0 \rightarrow N_A - W_{A_y} = 0 \rightarrow N_A = 83.24\,N \nwarrow$

1 pt

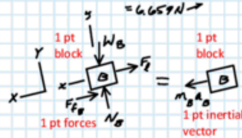
$+\swarrow \Sigma F_x = m_A a_A \rightarrow F_\ell + W_{Ax} - F_{f_A} = m_A a_A$

1 pt

$F_\ell + 83.24 - 6.659 = 12 a_A$ ①

$F_{f_A} = \mu_A N_A = 0.08(83.24)$

$= 6.659\,N \nearrow$



@B

1 pt $+\nwarrow \Sigma F_y = m_B a_{B_y} = 0 \rightarrow N_B - W_{B_y} = 0 \rightarrow N_B = 41.62\,N \nwarrow$

1 pt $+\swarrow \Sigma F_x = m_B a_B \rightarrow W_{Bx} - F_{f_B} - F_\ell = m_B a_B$

$41.62 - 11.65 - F_\ell = 6 a_B$ ②

1 pt $a_A = a_B = a$

① $\rightarrow F_\ell - 12a = -76.58$

② $\rightarrow F_\ell + 6a = 29.97$

2 eqn & 2 unknowns

$F_\ell = 12a - 76.58$

$12a - 76.58 + 6a = 29.97 \rightarrow 18a = 106.6$

$\boxed{a = 5.92\,m/s^2 \swarrow}$ (b)

1 pt value
1 pt units
1 pt direction
1 pt box or oval

$\boxed{F_\ell = -5.55\,N \text{ compression}}$ (a)

1 pt value
1 pt units
1 pt compression
1 pt box or oval

$F_{f_B} = \mu_B N_B = 0.28(41.62)$

$= 11.65\,N \nearrow$

$W_A = m_A g = 117.7\,N$

$W_B = m_B g = 58.86\,N$

$W_x = W_y = W \cos 45°$

$W_{Ax} = 83.24\,N$

$W_{Ay} = 83.24\,N$

$W_{Bx} = 41.62\,N$

$W_{By} = 41.62\,N$

Since forces do not vary with time, a = const.

2 pt $s = \cancel{s_0}^{\,0} + \cancel{v_0 t}^{\,0} + \frac{1}{2} a_c t^2 \rightarrow t = \left(\frac{2s}{a_c}\right)^{\frac{1}{2}}$

$t = \left[\frac{2(0.5)}{5.919}\right]^{\frac{1}{2}} = \boxed{0.411\text{ sec}}$ (c)

1 pt value
1 pt units
1 pt box or oval