Chapter 7
Response of First-Order *RL* and *RC* Circuits

Learning Goals for Chapter 7

- Be able to calculate the complete response for first-order *RL* and *RC* circuits with constant inputs

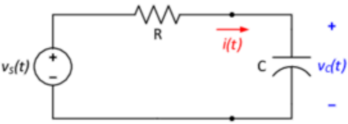
Complete Response of RC and RL Circuits

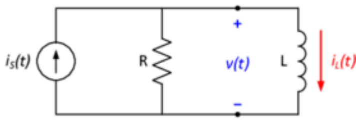
- Voltages and currents in circuits that contain capacitors and inductors are solutions to differential equations.

- The order of the differential equation is usually equal to the total number of capacitors and inductors in the circuit.

- Circuits that contain only one capacitor or inductor can be represented by a first-order differential equation and are called first-order circuits.

- All first-order circuits are equivalent to one of the following:





$$v_C(t) + RC\frac{dv_C(t)}{dt} = v_S(t)$$

$$\frac{dv_C(t)}{dt} + \frac{v_C(t)}{RC} = \frac{v_S(t)}{RC}$$

$$i_L(t) + \frac{L}{R}\frac{di_L(t)}{dt} = i_S(t)$$

$$\frac{di_L(t)}{dt} + \frac{R}{L}i_L(t) = \frac{R}{L}i_S(t)$$

- These first-order differential equations have the same form:

$$\frac{dx(t)}{dt} + \frac{x(t)}{\tau} = \frac{y(t)}{\tau}$$

RC Circuit

$y(t) = v_S(t)$

$x(t) = v_C(t)$

$\tau = RC$

RL Circuit

$y(t) = i_S(t)$

$x(t) = i_L(t)$

$\tau = \frac{L}{R}$

- The constant τ is called the time constant.

- The solution to this first-order differential equation can be expressed in several equivalent forms:

  complete response = transient response + steady-state response

  complete response = natural response + forced response

  complete response = homogeneous response + particular solution

- We will only consider first-order circuits with constant inputs.

- Solution approach:
  - Determine the initial condition of the energy storage element.
  - Determine the steady-state or forced response.
  - Add the natural response to the forced response to obtain the complete response. Use the initial condition to resolve the constant in the natural response.

Constant Input

$$y(t) = M = x(\infty)$$

Then:

$$\frac{dx(t)}{dt} + \frac{x(t)}{\tau} = \frac{M}{\tau}$$

$$\frac{dx(t)}{dt} = \frac{M - x(t)}{\tau}$$

$$\frac{dx(t)}{x(t) - M} = -\frac{dt}{\tau}$$

$$\int \frac{dx}{x - M} = -\frac{1}{\tau}\int dt$$

$$\ln(x - M) = -\frac{t}{\tau} + D$$

$$e^{\ln(x-M)} = e^{\frac{-t}{\tau}+D}$$

$$\mathrm{x} - \mathrm{M} = Ke^{\frac{-t}{\tau}} \qquad \text{where} \qquad K = e^{D}$$

So the complete response is:

$$x(t) = M + Ke^{\frac{-t}{\tau}}$$

At time $t = 0$:

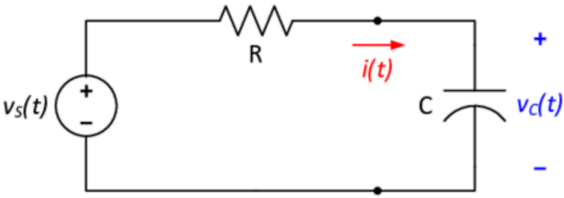
$$x(0) = M + K$$

$$K = x(0) - M = x(0) - x(\infty)$$

Finally:

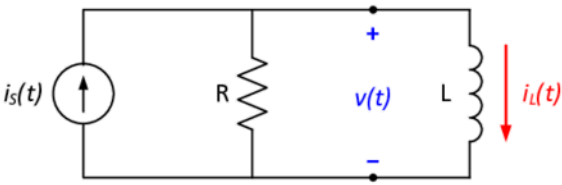
$$x(t) = x(\infty) + [x(0) - x(\infty)]e^{\frac{-t}{\tau}}$$

- For the original RC circuit with $v_S(t) = M = x(\infty) = V_{OC}$:



$$v_C(t) = V_{OC} + [v_C(0) - V_{OC}]e^{\frac{-t}{RC}}$$

- For the original RL circuit with $i_S(t) = M = x(\infty) = I_{SC}$:



$$i_L(t) = I_{SC} + [i_L(0) - I_{SC}]e^{\frac{-R}{L}t}$$

- These results can be shifted to time $t = t_0$:

$$v_C(t) = V_{OC} + [v_C(t_0) - V_{OC}]e^{\frac{-(t-t_0)}{RC}} \qquad \text{for } t > t_0$$

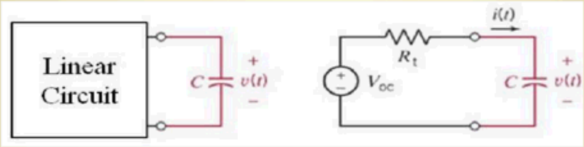
$$i_L(t) = I_{SC} + [i_L(t_0) - I_{SC}]e^{\frac{-R}{L}(t-t_0)} \qquad \text{for } t > t_0$$

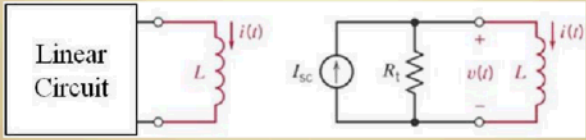
First-order Circuits

- Circuits that contain only one energy storage element (a capacitor or an inductor) and resistors.
- Circuits may be energized by voltage sources and/or current sources.
- We will consider only circuits with constant (DC) voltage or current sources.

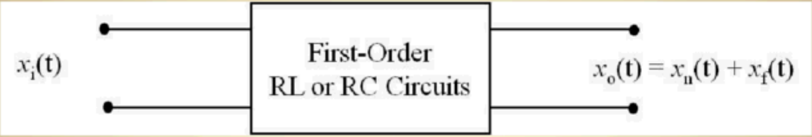
First-order Circuits

- Any first-order circuit can be reduced to either a Thévenin equivalent or a Norton equivalent circuit of the following forms:





First-order Circuits



$$x_0(t) = x_n(t) + x_f(t)$$

- $x_0(t)$: complete response
- $x_n(t)$: natural, transient or homogeneous response
- $x_f(t)$: forced, steady-state or particular response

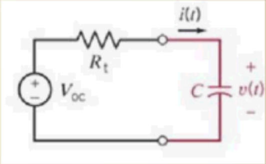
Complete Response to a Constant Input

- The complete response of the circuit is

$$x(t) = x(\infty) + \left[x(0) - x(\infty)\right]e^{-t/\tau}$$

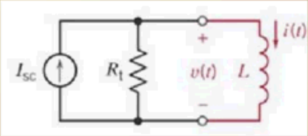
- For a *RC* circuit (***find Thévenin Equivalent Circuit***)

$$v_C(t) = v_{oc} + \left[v_C(0) - v_{oc}\right]e^{-t/\tau} \quad where \quad \tau = R_{th}C$$



- For a *RL* circuit (***find Norton Equivalent Circuit***),

$$i_L(t) = i_{sc} + \left[i_L(0) - i_{sc}\right]e^{-t/\tau} \quad where \quad \tau = \frac{L}{R_{th}}$$

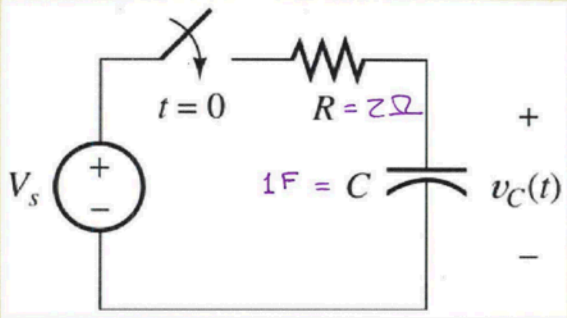


Example 8.7.2 – *RC* Circuit

- Find the expression for the voltage across the capacitor when $v_c(0^-) = 0$.

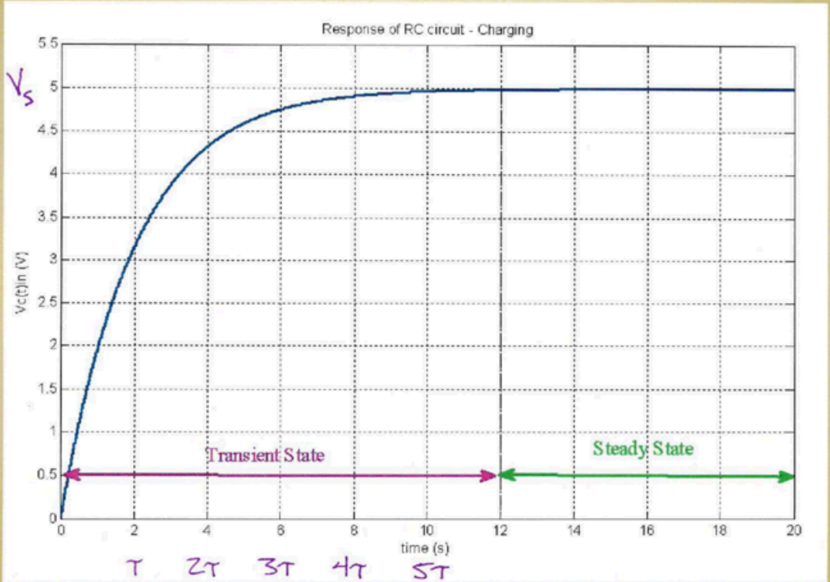
5V =

1F =



$\tau = RC$

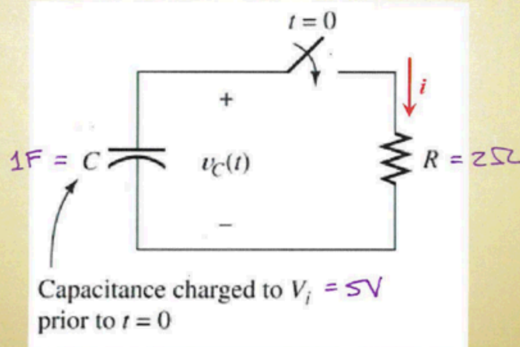
Capacitor



Response of RC circuit - Charging
Vc(t)in (V)
time (s)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
0
2
4
6
8
10
12
14
16
18
20
Vs
Transient State
Steady State
τ
2τ
3τ
4τ
5τ

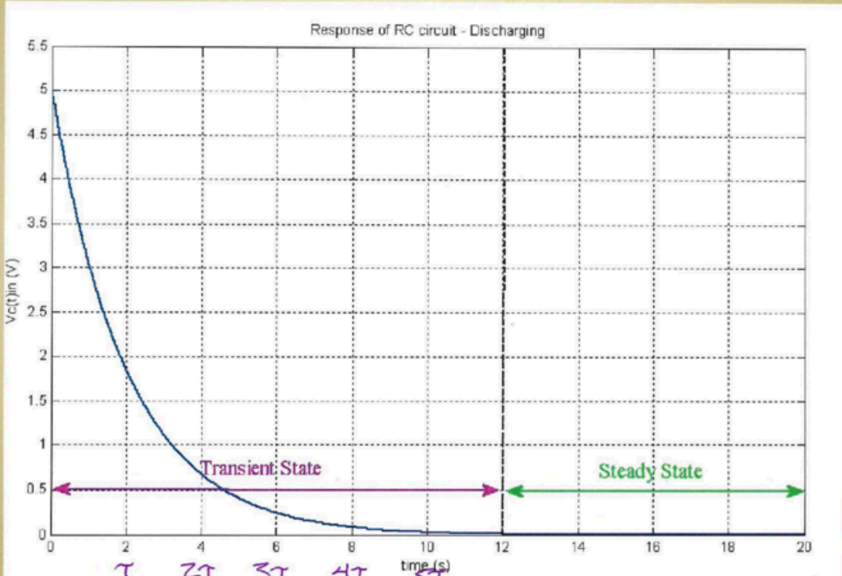
Example 8.7.1 – Discharge of a Capacitance through a Resistance

- Find the natural response of the voltage across the capacitor when the switch is closed at $t = 0$. Initially, the capacitor is charged to a voltage V_i



$\tau = RC$

Capacitor


Response of RC circuit - Discharging
Vc(t)in (V)
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
2
4
6
8
10
12
14
16
18
20
time (s)
Transient State
Steady State
τ
2τ
3τ
4τ
5τ

Example 1a

$$v_c(t) = V_{oc} + [v_c(0) - V_{oc}]e^{-\frac{t}{RC}}$$

$$V_{oc} = V_s = 5V$$
$$R = 2\Omega$$
$$C = 1F$$
$$v_c(0) = 0$$

$$v_c(t) = 5 + [0 - 5]e^{-\frac{t}{2}} = 5 - 5e^{-\frac{t}{2}} \; V$$

Example 1b

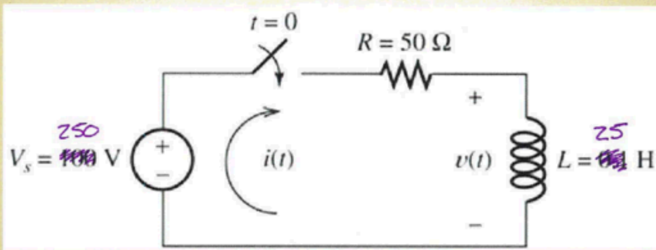
$$v_c(t) = V_{oc} + [v_c(0) - V_{oc}]e^{-\frac{t}{RC}}$$

$$V_{oc} = 0$$
$$R = 2\Omega$$
$$C = 1F$$
$$v_c(0) = 5V$$

$$v_c(t) = 0 + [5 - 0]e^{-\frac{t}{2}} = 5e^{-\frac{t}{2}} \; V$$

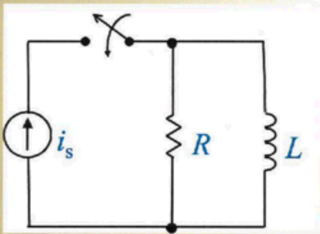
Example 8.7.3 – *RL* Circuit

- Find the express of the current.

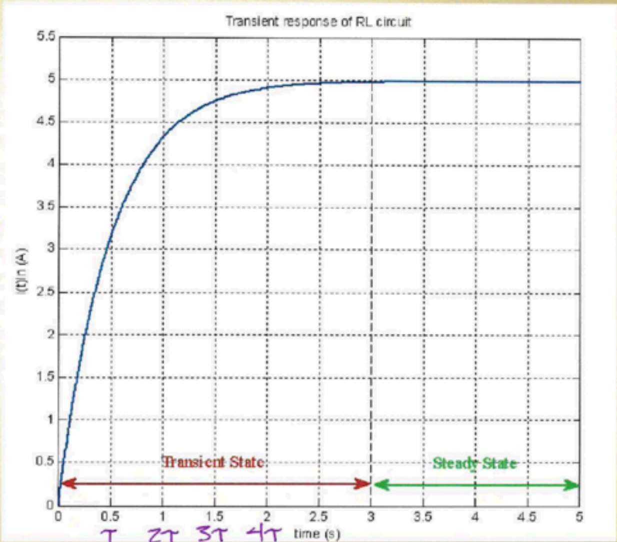


$$\tau = \frac{L}{R}$$

Inductor

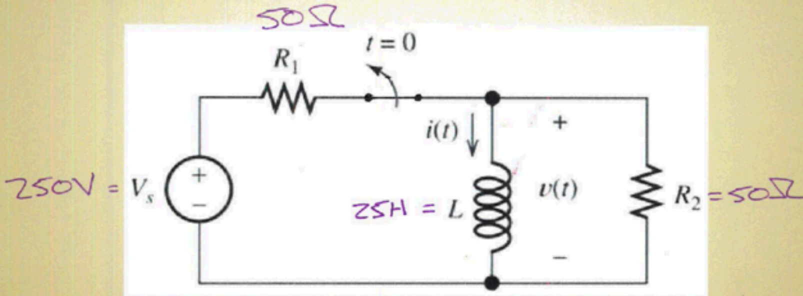


$$\tau = \frac{L}{R}$$



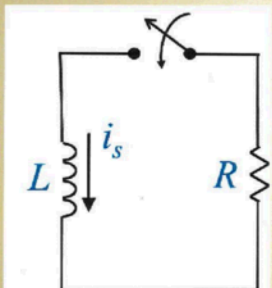
Example 8.7.4 – *RL* Circuit

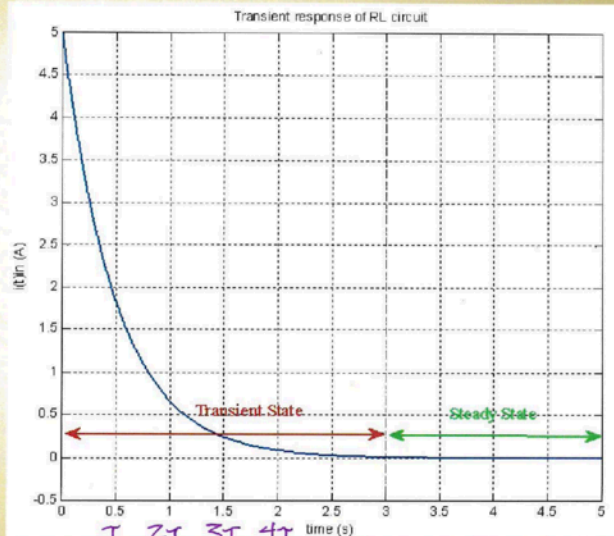
- Find the express of the current



$\tau = \frac{L}{R}$

Inductor





$$\tau = \frac{L}{R}$$

Example 2a

$$i_L(t) = I_{sc} + \left[i_L(0) - I_{sc}\right]e^{-\frac{R}{L}t}$$

$$I_{sc} = \frac{250}{50} = 5\,A$$

$$R = 50\,\Omega$$

$$L = 25\,H$$

$$i_L(0) = 0$$

$$i_L(t) = 5 + [0-5]e^{-2t} = 5 - 5e^{-2t}\,A$$

Example 2b

$$i_L(t) = I_{sc} + \left[i_L(0) - I_{sc}\right]e^{-\frac{R}{L}t}$$

$$I_{sc} = 0$$

$$R = 50\,\Omega$$

$$L = 25\,H$$

$$i_L(0) = \frac{250}{50} = 5\,A$$

$$i_L(t) = 0 + [5-0]e^{-2t} = 5e^{-2t}\,A$$

$e^{-1} = 0.3679$ $\quad$ $1-e^{-1} = 0.6321$

$e^{-2} = 0.1353$ $\quad$ $1-e^{-2} = 0.8647$

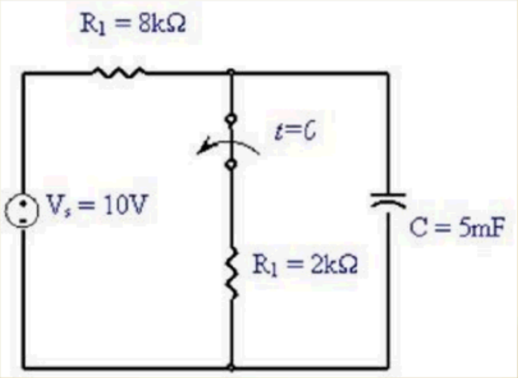
$e^{-3} = 0.0498$ $\quad$ $1-e^{-3} = 0.9502$

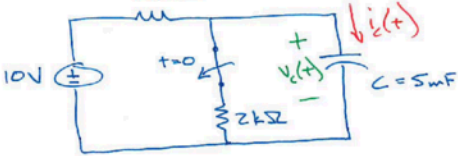
$e^{-4} = 0.0183$ $\quad$ $1-e^{-4} = 0.9817$

$e^{-5} = 0.0067$ $\quad$ $1-e^{-5} = 0.9933$

Example 8.7.5 – RC Circuit

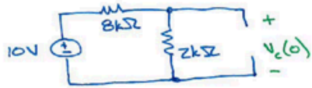
- Find the voltage across the capacitor when the switch is open at $t = 0$.





- Determine the initial condition.

Prior to $t=0$ the circuit is assumed to be in the steady state so $i_c = C\frac{dv_c(t)}{dt} = 0$.



$$v_c(0) = \frac{2k\Omega}{8k\Omega + 2k\Omega}(10V) = 2V$$

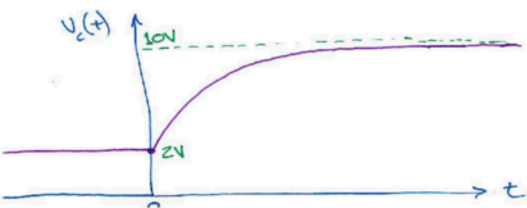
- After the switch is opened the steady state will be:

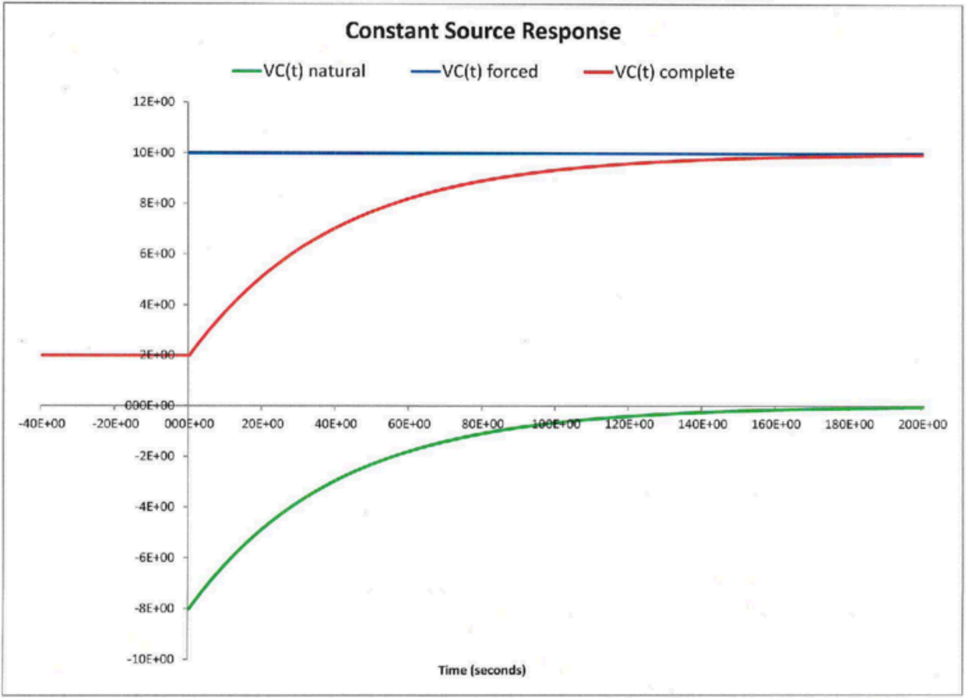
$$V_{oc} = 10V = v_c(\infty)$$

$$\tau = RC = 40 \text{ sec}$$

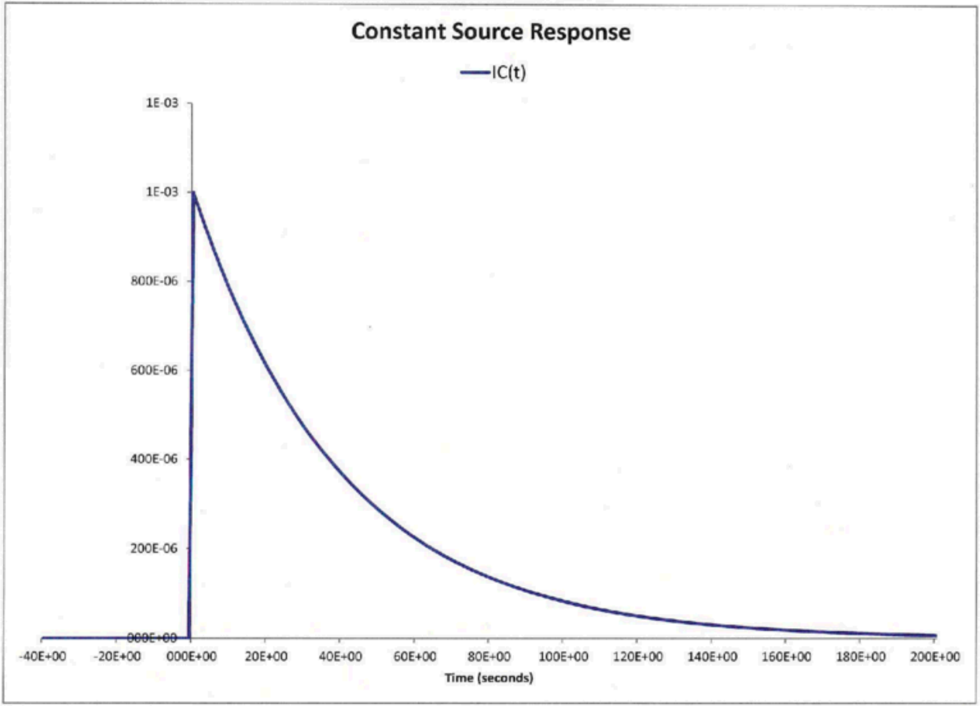
$$v_c(t) = V_{oc} + \{v_c(0) - V_{oc}\} e^{-\frac{t}{\tau}}$$

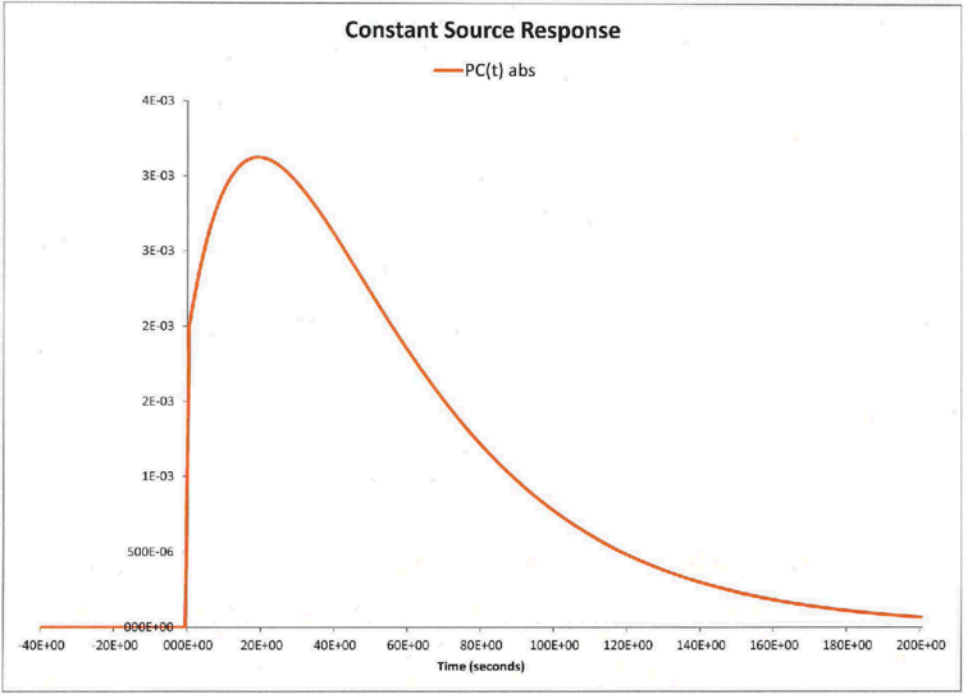
$$= 10 + (2-10)e^{-\frac{t}{40}} = 10 - 8e^{-\frac{t}{40}} V$$



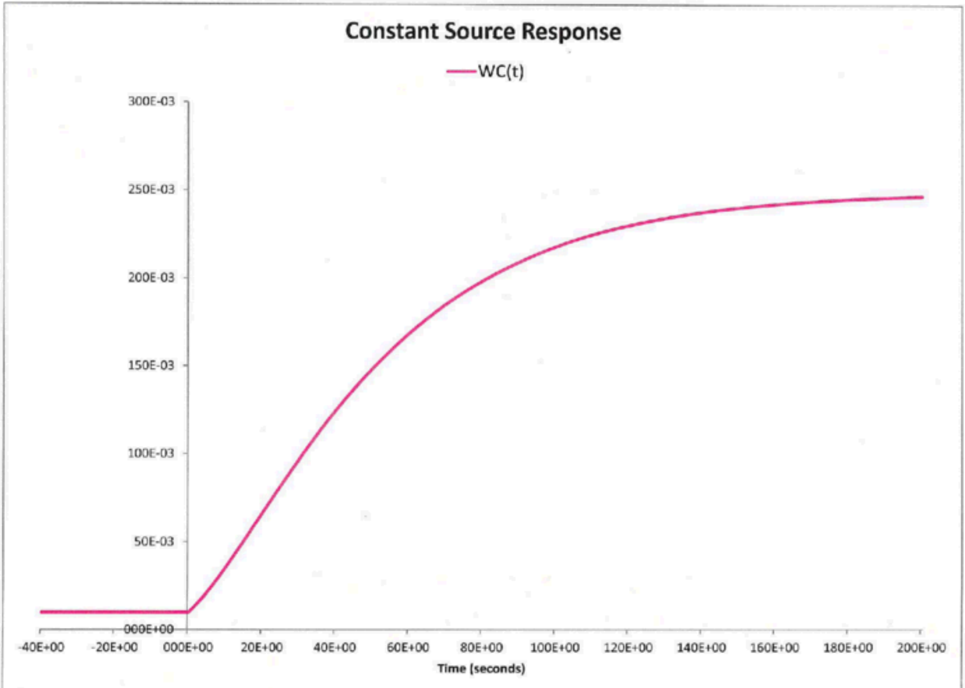


Constant Source Response
VC(t) natural
VC(t) forced
VC(t) complete
12E+00
10E+00
8E+00
6E+00
4E+00
2E+00
000E+00
-2E+00
-4E+00
-6E+00
-8E+00
-10E+00
-40E+00
-20E+00
000E+00
20E+00
40E+00
60E+00
80E+00
100E+00
120E+00
140E+00
160E+00
180E+00
200E+00
Time (seconds)


Constant Source Response
IC(t)
1E-03
1E-03
800E-06
600E-06
400E-06
200E-06
000E+00
-40E+00
-20E+00
000E+00
20E+00
40E+00
60E+00
80E+00
100E+00
120E+00
140E+00
160E+00
180E+00
200E+00
Time (seconds)



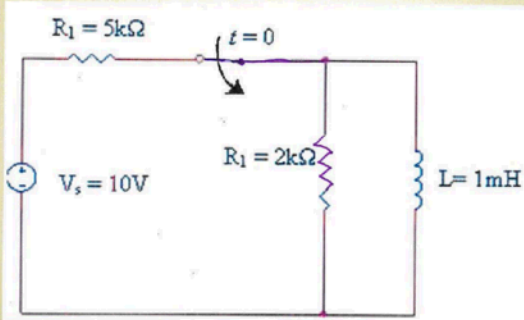
Constant Source Response
PC(t) abs
4E-03
3E-03
3E-03
2E-03
2E-03
1E-03
500E-06
000E+00
-40E+00
-20E+00
000E+00
20E+00
40E+00
60E+00
80E+00
100E+00
120E+00
140E+00
160E+00
180E+00
200E+00
Time (seconds)

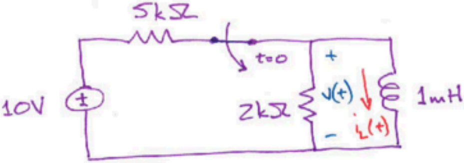


Constant Source Response
WC(t)
300E-03
250E-03
200E-03
150E-03
100E-03
50E-03
000E+00
-40E+00
-20E+00
000E+00
20E+00
40E+00
60E+00
80E+00
100E+00
120E+00
140E+00
160E+00
180E+00
200E+00
Time (seconds)

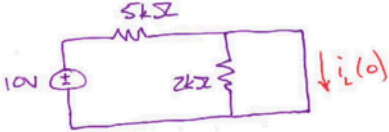
Example 8.7.6 – *RL* Circuit

- Find the expression for the current going through the inductor after the switch is open





Determine the initial inductor current.



$$i_L(0) = \frac{10V}{5k\Omega} = 2mA$$

After the switch is opened

$$I_{sc} = 0$$
$$R = 2k\Omega$$
$$L = 1mH$$
$$i_L(0) = 2mA$$

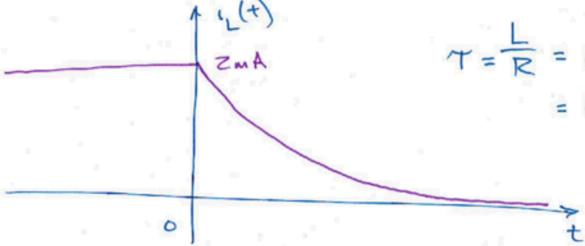
$$i_L(t) = I_{sc} + \left[i_L(0) - I_{sc}\right] e^{-\frac{R}{L}t}$$

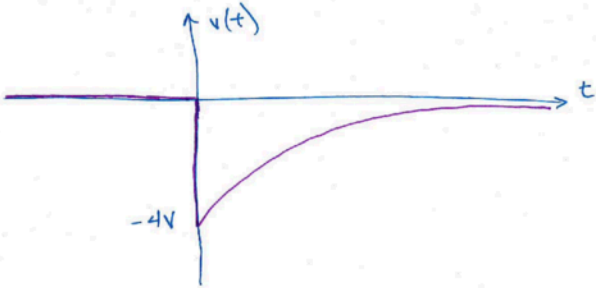
$$= 0 + \left[.002 - 0\right] e^{-\frac{2000}{.001}t}$$

$$= .002\, e^{-2\times10^{6}t} \text{ A}$$

$$v(t) = L\frac{di_L(t)}{dt} = (.001)(.002)\left(-2\times10^{6} e^{-2\times10^{6}t}\right) = -4e^{-2\times10^{6}t}\text{ V}$$

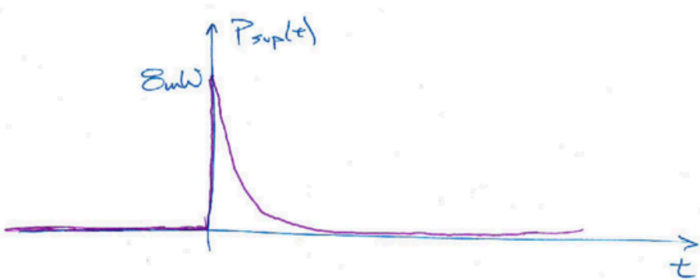
$$\tau = \frac{L}{R} = 0.5 \times 10^{-6}$$
$$= 0.5\,\mu\text{sec}$$





$$P_{abs}(t) = i_L(t)v(t) = -.008\,e^{-4\times10^{6}t}\ \text{W}$$

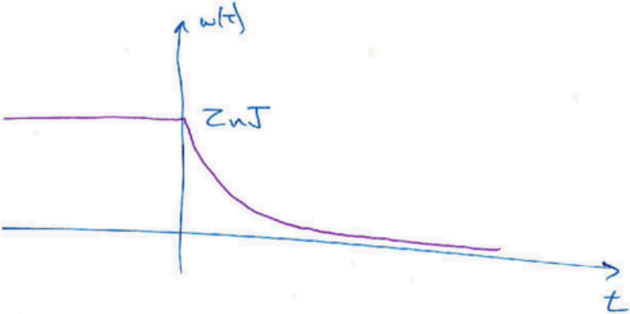
$$P_{sup}(t) = -P_{abs}(t) = .008\,e^{-4\times10^{6}t}\ \text{W}$$

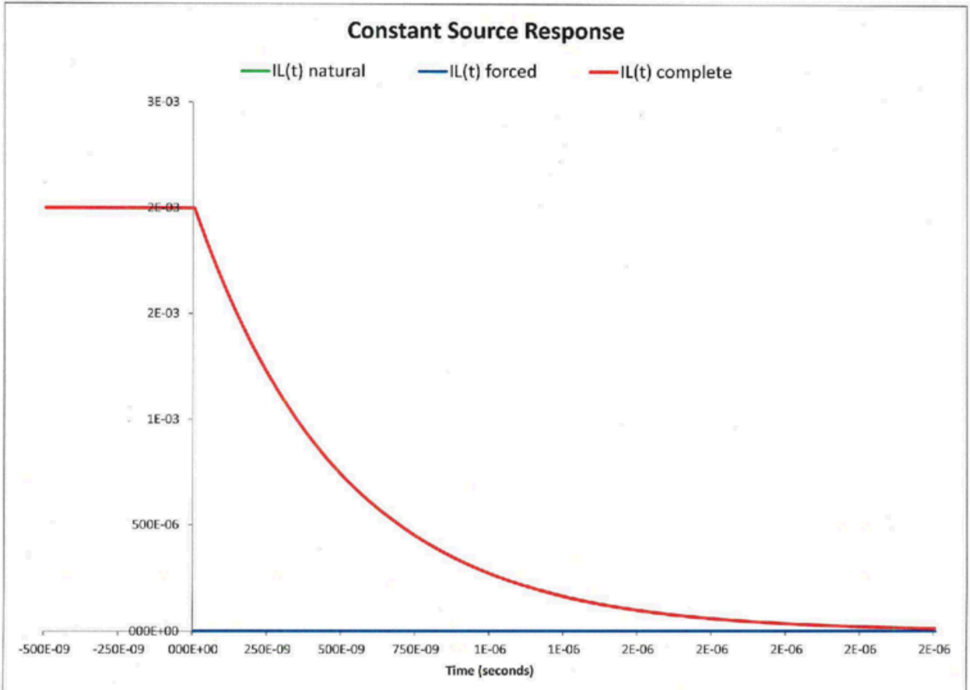


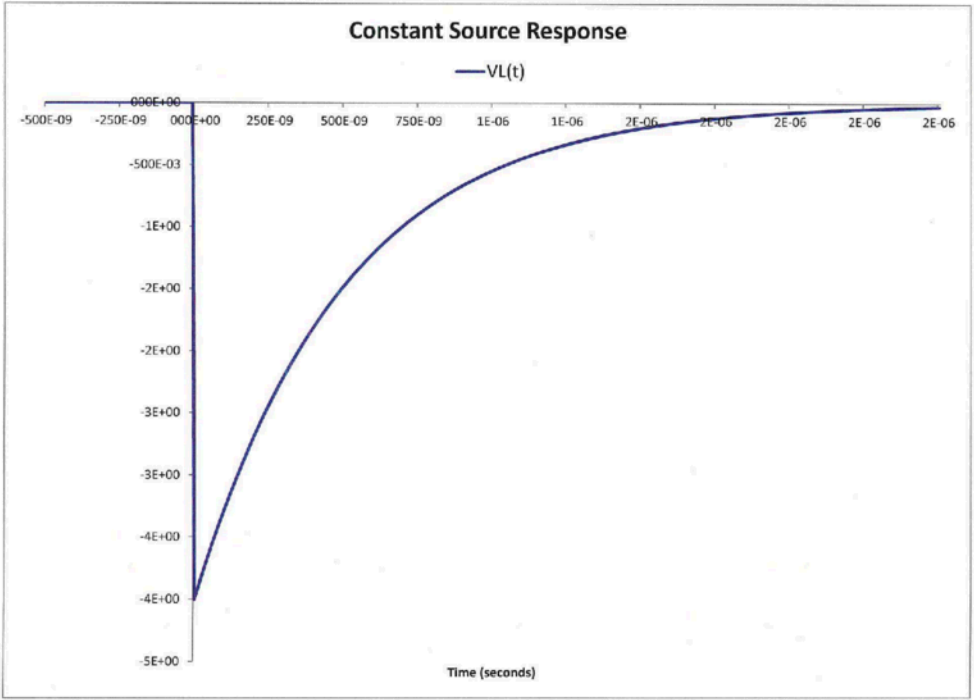
$$w(t) = \frac{1}{2} L i_L^2(t)$$

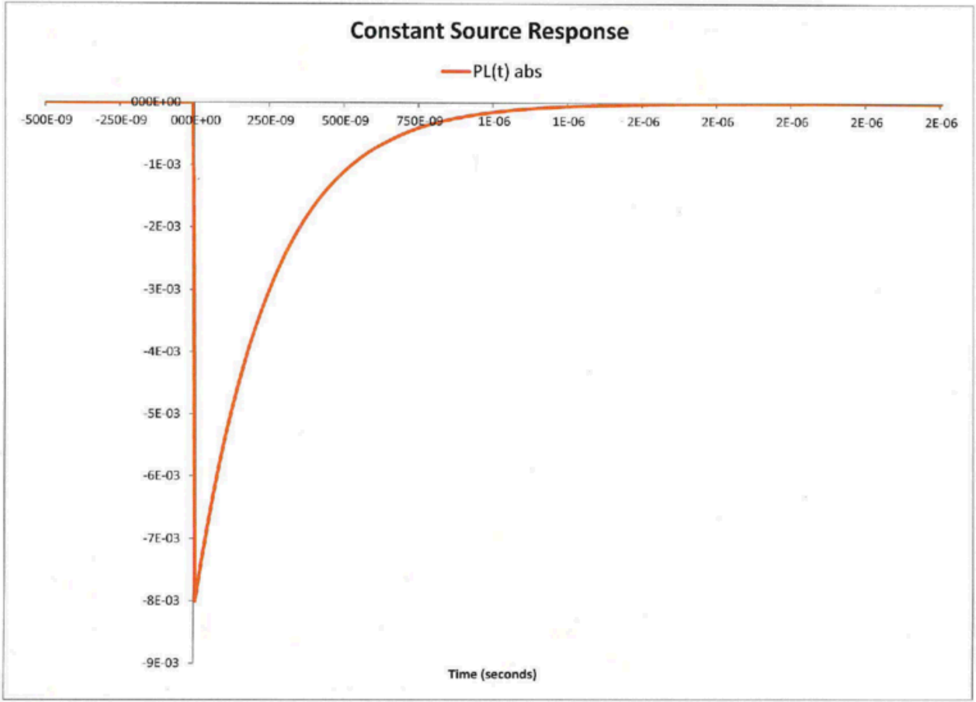
$$= \frac{.001}{2}\left(.002 e^{-2\times10^6 t}\right)^2$$

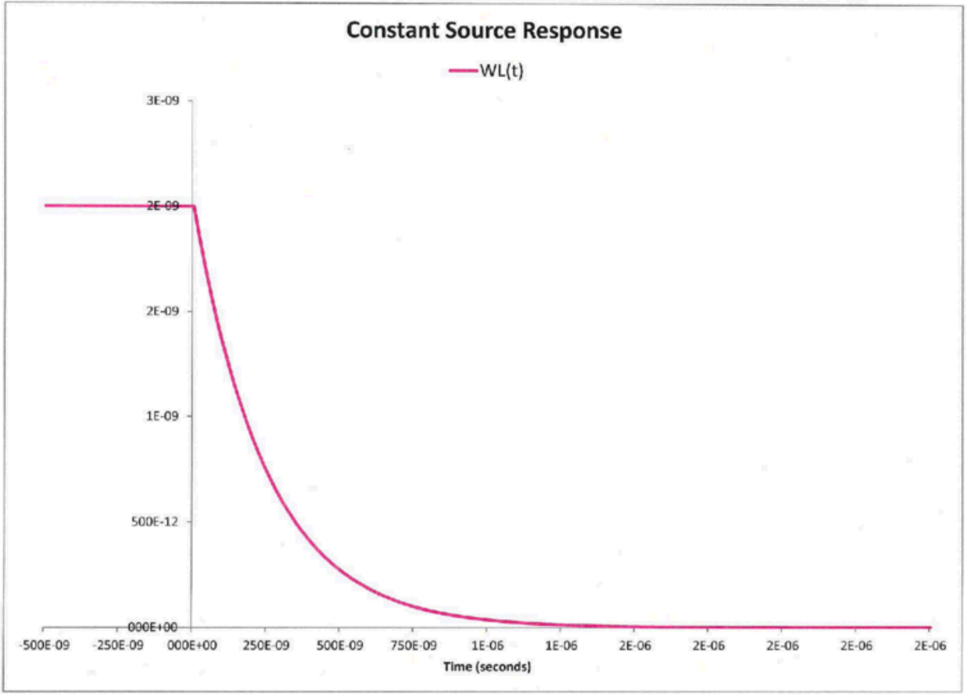
$$= 2\times10^{-9} e^{-4\times10^6 t} \ \text{J}$$



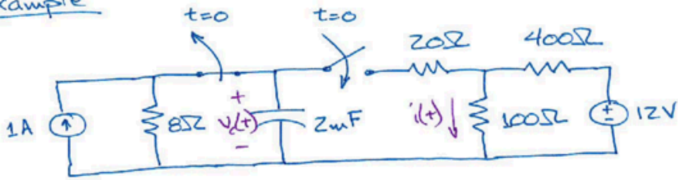

Constant Source Response
IL(t) natural
IL(t) forced
IL(t) complete
3E-03
2E-03
2E-03
1E-03
500E-06
000E+00
-500E-09
-250E-09
000E+00
250E-09
500E-09
750E-09
1E-06
1E-06
2E-06
2E-06
2E-06
2E-06
2E-06
Time (seconds)


Constant Source Response
VL(t)
000E+00
-500E-03
-1E+00
-2E+00
-2E+00
-3E+00
-3E+00
-4E+00
-4E+00
-5E+00
-500E-09
-250E-09
000E+00
250E-09
500E-09
750E-09
1E-06
1E-06
2E-06
2E-06
2E-06
2E-06
2E-06
Time (seconds)


Constant Source Response
PL(t) abs
000E+00
-500E-09
-250E-09
000E+00
250E-09
500E-09
750E-09
1E-06
1E-06
2E-06
2E-06
2E-06
2E-06
2E-06
-1E-03
-2E-03
-3E-03
-4E-03
-5E-03
-6E-03
-7E-03
-8E-03
-9E-03
Time (seconds)

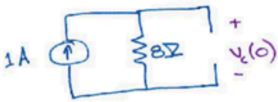

Constant Source Response
WL(t)
3E-09
2E-09
2E-09
1E-09
500E-12
000E+00
-500E-09
-250E-09
000E+00
250E-09
500E-09
750E-09
1E-06
1E-06
2E-06
2E-06
2E-06
2E-06
2E-06
Time (seconds)

Example



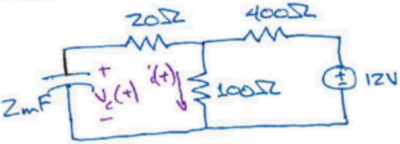
Determine $i(t)$.

- Determine the initial condition for the capacitor voltage $v_c(0)$.



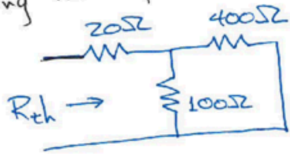
$$v_c(0) = 1A(8\Omega) = 8V$$

- After the switches are thrown the circuit is:

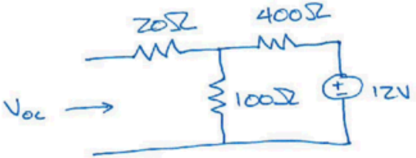


$$v_c(0) = 8V$$

- Find the Thevenin equivalent of the circuit facing the capacitor.

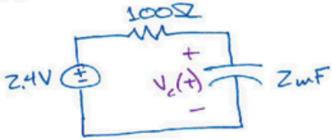


$$R_{th} = 20 + 100 \| 400$$
$$= 20 + 80$$
$$= 100\Omega$$



$$V_{oc} = \frac{100}{100+400}(12V)$$
$$= 2.4V$$

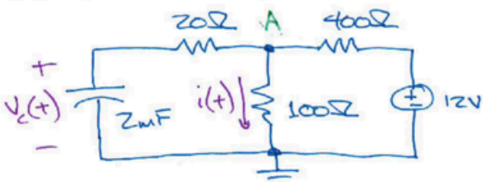
- So our circuit is:



$V_{oc} = 2.4V$
$R = 100\Omega$
$C = 2mF$
$\tau = RC = 0.2\text{ sec}$
$v_c(0) = 8V$

$$v_c(t) = V_{oc} + \left[v_c(0) - V_{oc}\right]e^{-t/\tau}$$
$$= 2.4 + (8-2.4)e^{-t/.2}$$
$$= 2.4 + 5.6e^{-5t}\ V$$

- Now to find $i(t)$



Write a KCL equation at Node A:

$$\frac{v_c(t) - v_a(t)}{20} + \frac{12 - v_a(t)}{400} = \frac{v_a(t)}{100}$$

$$20\{v_c(t) - v_a(t)\} + 12 - v_a(t) = 4v_a(t)$$

$$20v_c(t) - 20v_a(t) + 12 - v_a(t) = 4v_a(t)$$

$$v_a(t) = \frac{20v_c(t) + 12}{25}$$

$$i(t) = \frac{v_a(t)}{100}$$

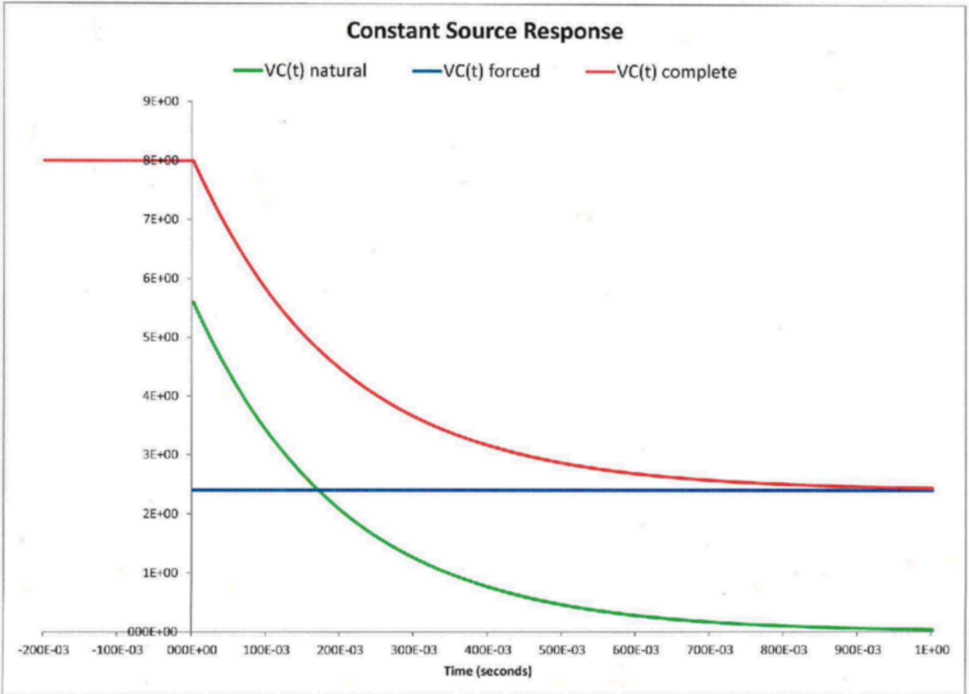
$$= \frac{20v_c(t) + 12}{25 \cdot 100}$$

$$= .008v_c(t) + .0048$$

$$= .008\{2.4 + 5.6e^{-5t}\} + .0048$$

$$= .024 + .0448e^{-5t} \text{ A}$$

$$= \underline{\underline{24 + 44.8e^{-5t} \text{ mA}}}$$

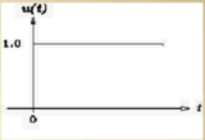


Constant Source Response
VC(t) natural
VC(t) forced
VC(t) complete
9E+00
8E+00
7E+00
6E+00
5E+00
4E+00
3E+00
2E+00
1E+00
000E+00
-200E-03
-100E-03
000E+00
100E-03
200E-03
300E-03
400E-03
500E-03
600E-03
700E-03
800E-03
900E-03
1E+00
Time (seconds)

Unit Step Function

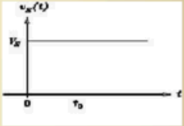
- Unit Step Function

$$u(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1 & \text{for } t \geq 0 \end{cases}$$



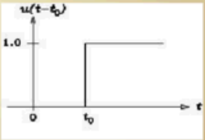
- Step Voltage Source

$$v_S(t) = V_S u(t) = \begin{cases} 0 & \text{for } t < 0 \\ \mathrm{V_S} & \text{for } t \geq 0 \end{cases}$$



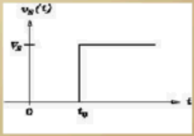
- Unit Step Function for $t_0 \neq 0$

$$u(t - t_0) = \begin{cases} 0 & \text{for } t < t_0 \\ 1 & \text{for } t \geq t_0 \end{cases}$$



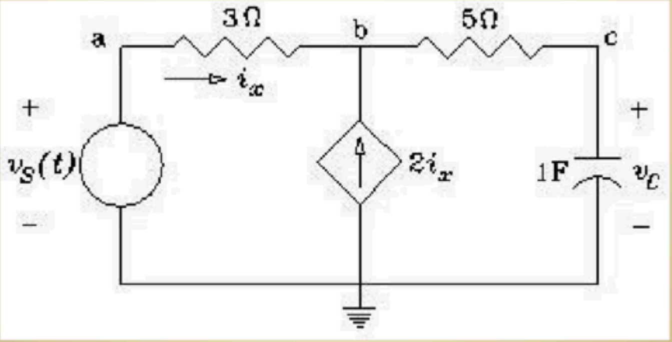
- Step Voltage Source for $t_0 \neq 0$

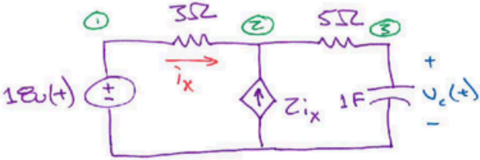
$$v_S(t) = V_S u(t - t_0) = \begin{cases} 0 & \text{for } t < t_0 \\ \mathrm{V_S} & \text{for } t \geq t_0 \end{cases}$$



Example 8.7.14 – *RC* Circuit with Dependent Source

- Consider the circuit shown with a controlled current source. The circuit is initially at rest. Find the complete response of the capacitor voltage. The source voltage is $v_S(t) = 18\ u(t)$.

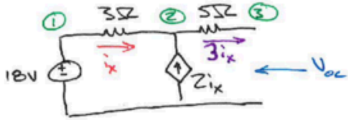




$v_c(0) = 0$

Find $v_c(t)$

• Determine a Thevenin equivalent.



KCL at Node 2:

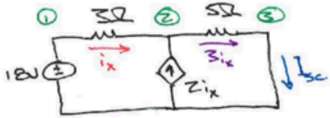
$$i_x + 2i_x = 3i_x$$

Because of the open circuit:

$$3i_x = 0$$

$$i_x = 0$$

$$V_{oc} = 18\,V$$



KCL at Node 2:

$$i_x + 2i_x = 3i_x$$

KVL around outer loop:

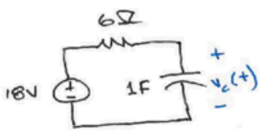
$$-18 + 3i_x + 5(3i_x) = 0$$

$$18i_x = 18$$

$$i_x = 1$$

$$I_{sc} = 3i_x = 3A$$

$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{18V}{3A} = 6\Omega$$



$v_c(0) = 0$

$V_{oc} = 18\,V$

$R = 6\Omega$

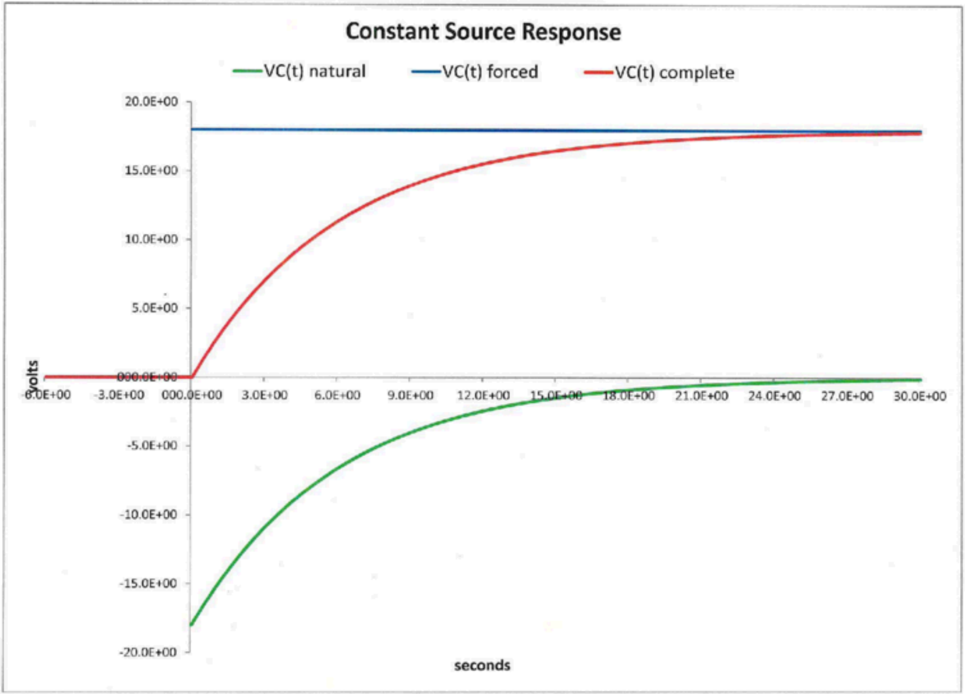
$C = 1\,F$

$\tau = RC = 6$ sec

• So,

$$v_c(t) = V_{oc} + [v_c(0) - V_{oc}]\,e^{-t/RC}$$

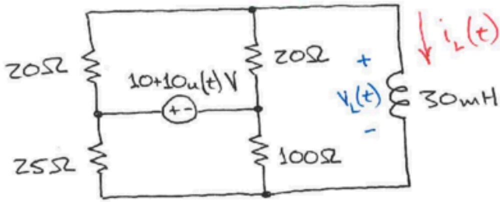
$$= \underline{\underline{18 - 18e^{-t/6}\ V}}$$



Constant Source Response
VC(t) natural
VC(t) forced
VC(t) complete
20.0E+00
15.0E+00
10.0E+00
5.0E+00
000.0E+00
-5.0E+00
-10.0E+00
-15.0E+00
-20.0E+00
volts
-6.0E+00
-3.0E+00
000.0E+00
3.0E+00
6.0E+00
9.0E+00
12.0E+00
15.0E+00
18.0E+00
21.0E+00
24.0E+00
27.0E+00
30.0E+00
seconds

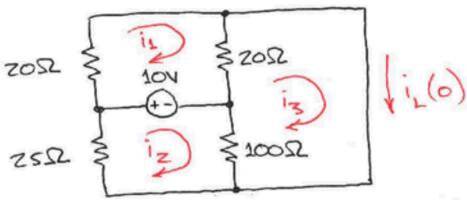
RL Example

Determine $i_L(t)$ and $v_L(t)$.



First find $i_L(0)$.

In the steady-state just prior to $t=0$:



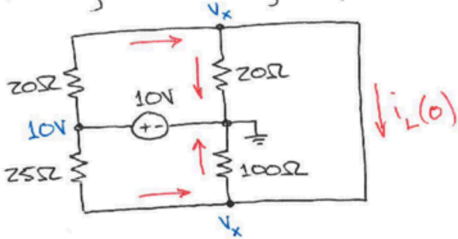
Mesh current equations:

$$20i_1 + 20(i_1 - i_3) - 10 = 0$$
$$25i_2 + 10 + 100(i_2 - i_3) = 0$$
$$20(i_3 - i_1) + 100(i_3 - i_2) = 0$$

$$\begin{bmatrix} 40 & 0 & -20 \\ 0 & 125 & -100 \\ -20 & -100 & 120 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -10 \\ 0 \end{bmatrix}$$

From which $i_3 = i_L(0) = -0.1\text{ A}$

Or using node voltage equations:



$$\frac{10-v_x}{20} = i_L(0) + \frac{v_x}{20}$$

$$\frac{10-v_x}{25} + i_L(0) = \frac{v_x}{100}$$

$$\longrightarrow$$

$$\frac{10-v_x}{20} - \frac{v_x}{20} = i_L(0)$$

$$\frac{10-v_x}{25} - \frac{v_x}{100} = -i_L(0)$$

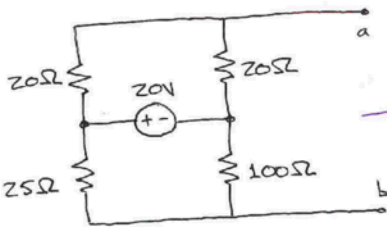
$$\frac{10-v_x}{20} - \frac{v_x}{20} + \frac{10-v_x}{25} - \frac{v_x}{100} = 0$$

From which:

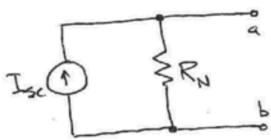
$$v_x = 6\text{V}$$

$$i_L(0) = \frac{10-6}{20} - \frac{6}{20} = -0.1\text{A} \quad (\text{as before})$$

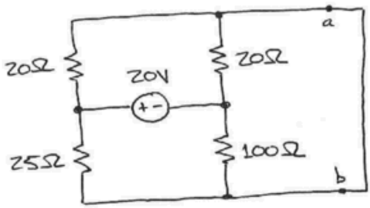
Next find the Norton equivalent for $t \geq 0$



$\longrightarrow$



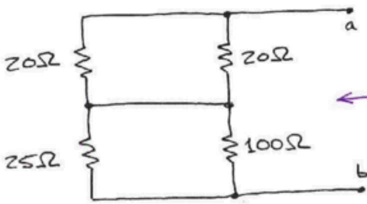
For I_{sc}:



This is the same circuit as the one for finding $i_L(0)$ except the voltage source is doubled (20V vs. 10V). Therefore:

$$I_{sc} = 2\, i_L(0) = -0.2\,\text{A}$$

For R_N:



$$R_{eq} = 20\Omega \| 20\Omega + 25\Omega \| 100\Omega$$

$$= 10\Omega + 20\Omega$$

$$= 30\Omega$$

So,

$$i_L(0) = -0.1\,A$$

$$I_{sc} = -0.2\,A$$

$$\tau = \frac{L}{R_N} = \frac{.03}{30} = 1\,ms$$

$$i_L(t) = I_{sc} + \left[i_L(0) - I_{sc}\right] e^{-t/\tau}$$

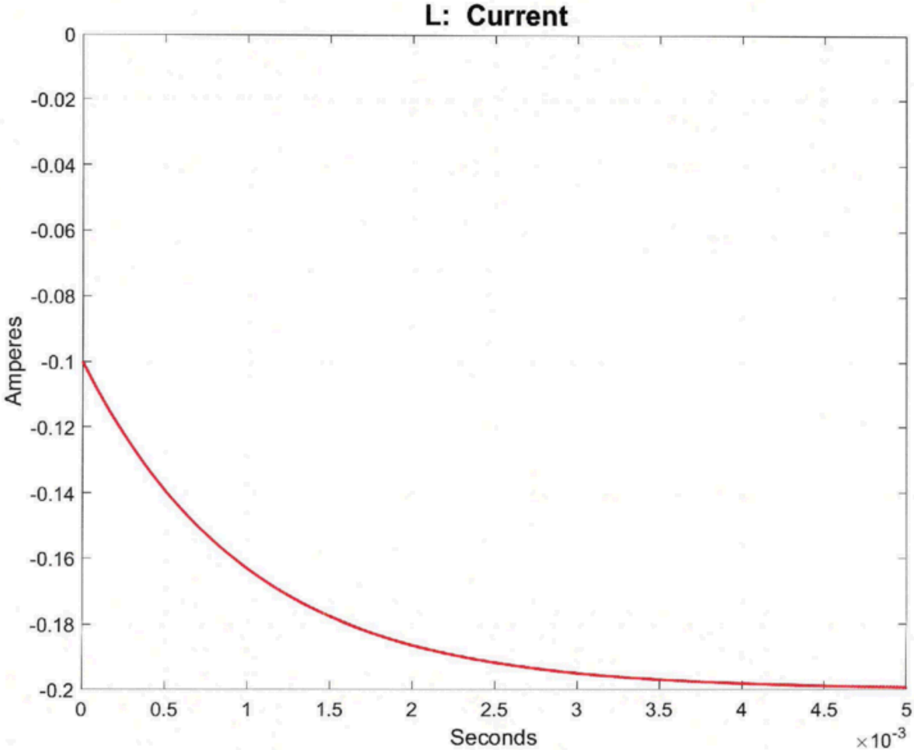
$$= -0.2 + \left[-0.1 - (-0.2)\right] e^{-t/.001}$$

$$= \underline{\underline{-0.2 + 0.1 e^{-1000t}\ A}}$$

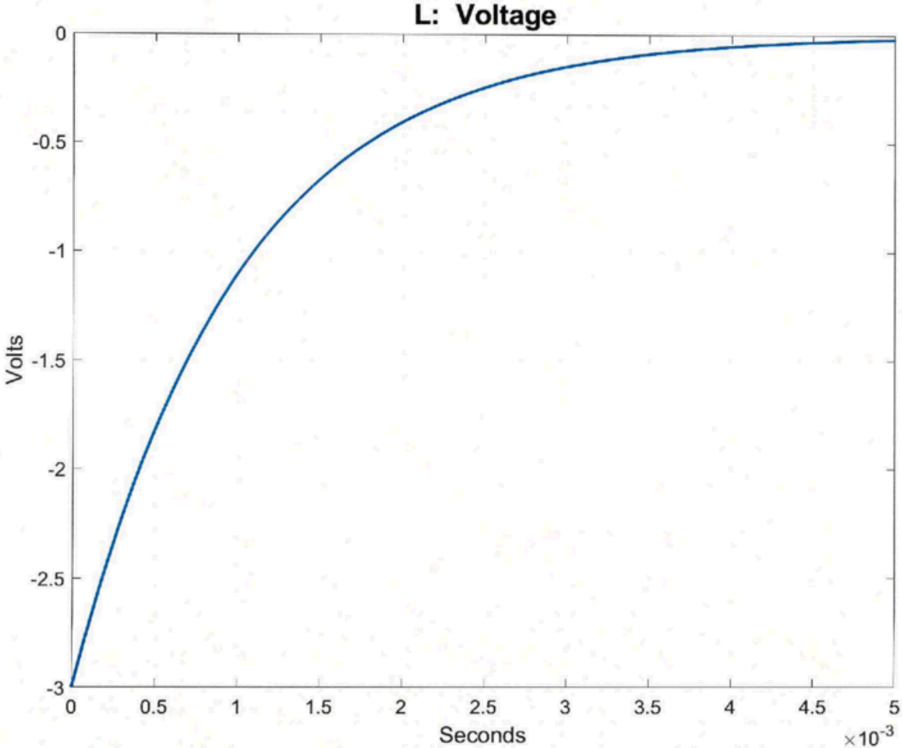
$$v_L(t) = L\frac{di_L}{dt}$$

$$= 0.03\,(0.1)(-1000)\,e^{-1000t}$$

$$= \underline{\underline{-3e^{-1000t}\ V}}$$

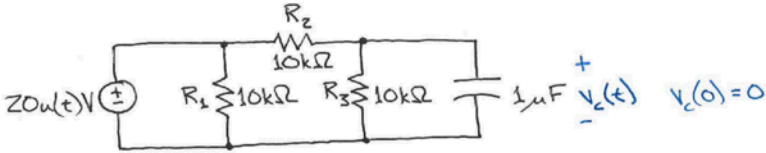


L: Current
Amperes
Seconds
×10^-3
0
-0.02
-0.04
-0.06
-0.08
-0.1
-0.12
-0.14
-0.16
-0.18
-0.2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5



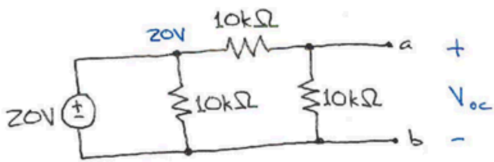
L: Voltage
0
-0.5
-1
-1.5
-2
-2.5
-3
Volts
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Seconds
×10^-3

Example



- Determine the capacitor voltage, current, power absorbed and stored energy.
- Determine the power absorbed by the resistors.
- Determine the power supplied by the source.
- Evaluate these expressions for $t = 10\,ms$.

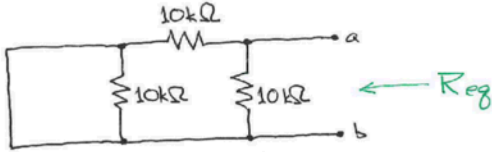
First find the Thevenin equivalent circuit facing the capacitor for $t \geq 0$:



By voltage division:

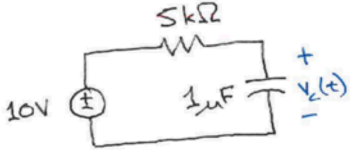
$$V_{oc} = \frac{10}{10+10}(20) = 10V$$

Since there are no dependent sources



$$R_{eq} = 10k\Omega \| 10k\Omega = 5k\Omega$$

The Thevenin equivalent is then:



$v_c(0) = 0$

$V_{oc} = 10V$

$\tau = RC = (5k\Omega)(1\mu F) = 5ms$

From which:

$$v_c(t) = V_{oc} + [v_c(0) - V_{oc}]e^{-t/\tau}$$

$$= 10 + [0 - 10]e^{-t/.005}$$

$$= \underline{\underline{10 - 10e^{-200t} V}}$$

$$i_c(t) = C\frac{dv_c(t)}{dt}$$

$$= 10^{-6}(-10)(-200)e^{-200t}$$

$$= \underline{\underline{2e^{-200t} mA}}$$

$$P_c(t) = v_c(t)\, i_c(t)$$

$$= \left(10 - 10e^{-200t}\right)\left(2e^{-200t}\right)$$

$$= \underline{\underline{20e^{-200t} - 20e^{-400t} \text{ mW}}}$$

$$w_c(t) = \frac{1}{2} C v_c^2(t)$$

$$= \frac{1}{2}\left(10^{-6}\right)\left(10 - 10e^{-200t}\right)^2$$

$$= \underline{\underline{50 - 100e^{-200t} + 50e^{-400t} \ \mu\text{J}}}$$

Now determine the power absorbed by the resistors:

$$P_{R_1} = \frac{V_s^2}{R_1} = \frac{20^2}{10^4} = \underline{\underline{40 \text{ mW}}}$$

$$P_{R_2}(t) = \frac{\left[V_s - v_c(t)\right]^2}{R_2}$$

$$= \frac{\left(10 + 10e^{-200t}\right)^2}{10^4}$$

$$= \underline{\underline{10 + 20e^{-200t} + 10e^{-400t} \text{ mW}}}$$

$$P_{R_3}(t) = \frac{v_c^2(t)}{R_3}$$

$$= \frac{\left(10 - 10e^{-200t}\right)^2}{10^4}$$

$$= \underline{\underline{10 - 20e^{-200t} + 10e^{-400t} \text{ mW}}}$$

For the power supplied by the source:

$$p_{V_s}(t) = p_{R_1} + p_{R_2}(t) + p_{R_3}(t) + p_C(t)$$

$$= \underline{\underline{60 + 20e^{-200t} \text{ mW}}}$$

Finally at $t = 10$ ms:

$v_C = 8.647$ V

$i_C = 270.6\ \mu$A

$p_C = 2.340$ mW (abs)

$p_{R_1} = 40$ mW (abs)

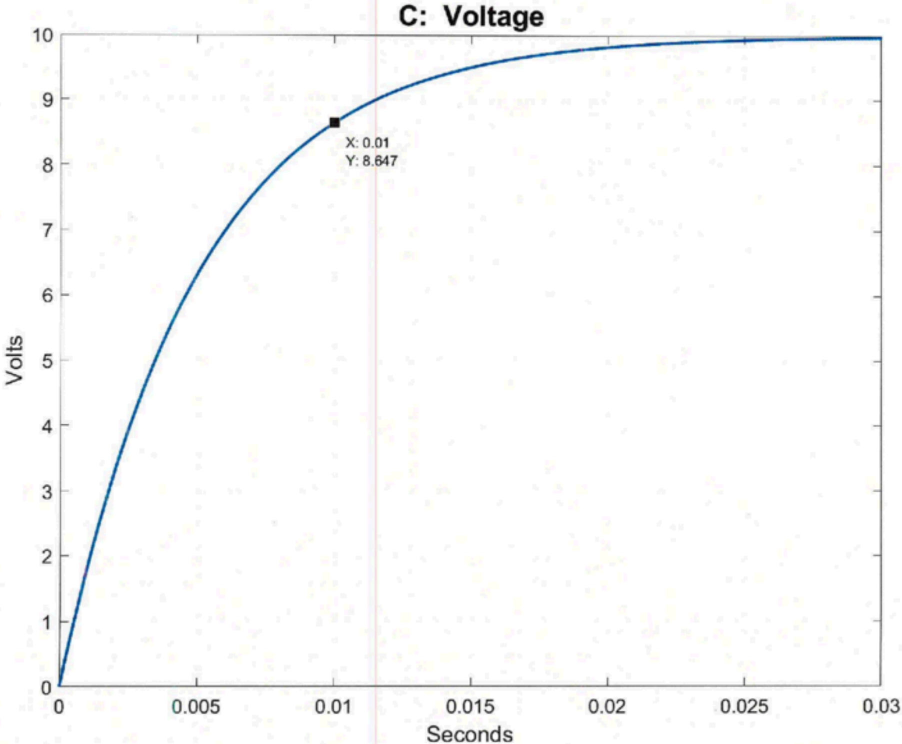
$p_{R_2} = 12.89$ mW (abs)

$p_{R_3} = 7.477$ mW (abs)

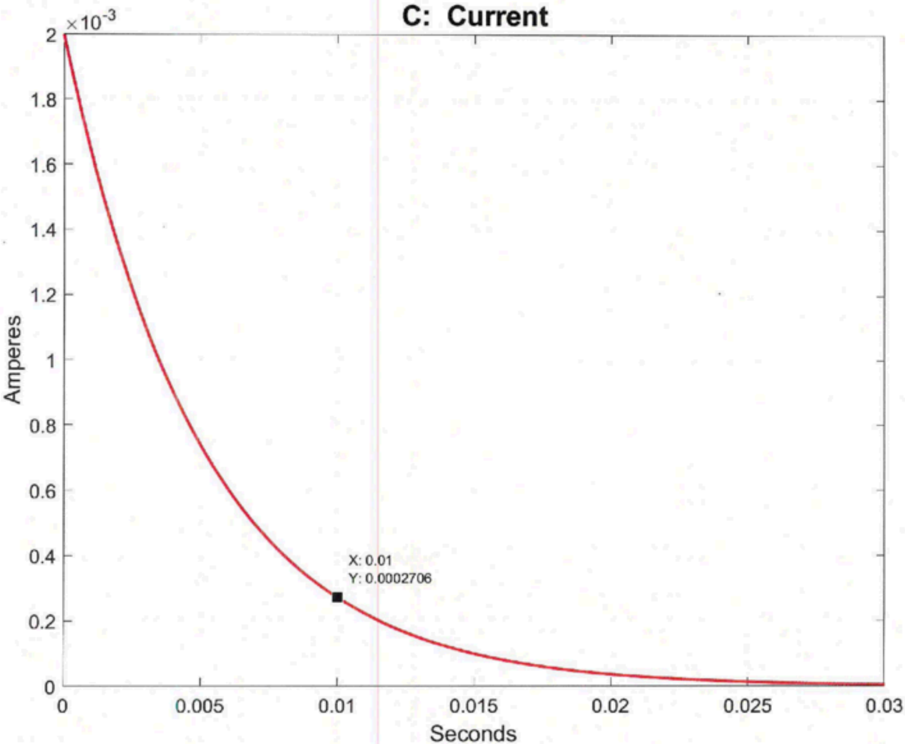
$p_{V_s} = 62.71$ mW (sup)

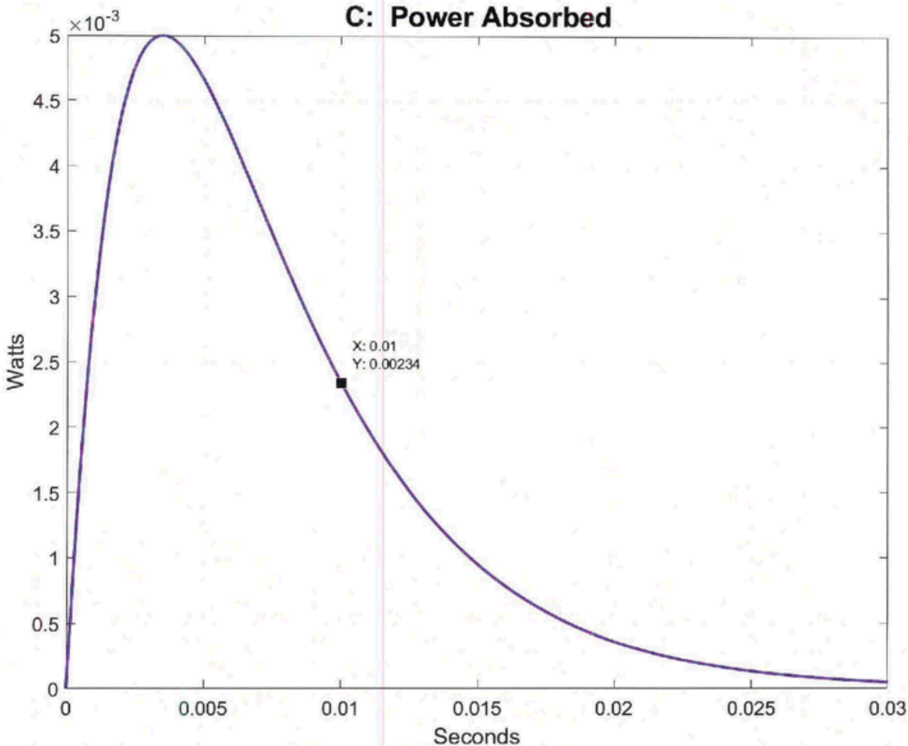
$w_C = 37.39\ \mu$J

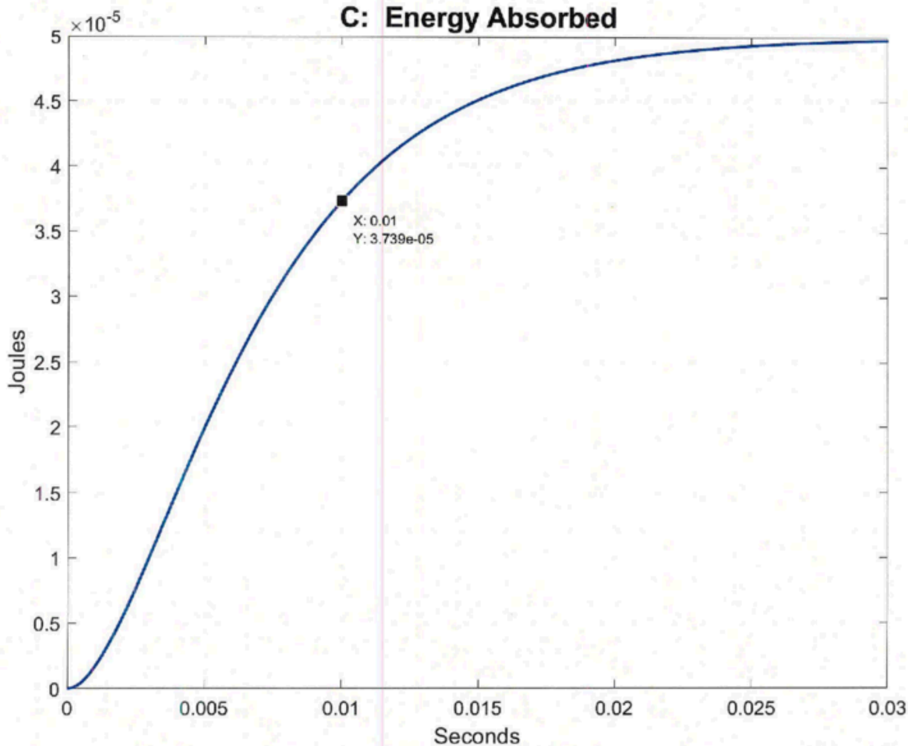
Note: $\sum p_{sup} \overset{\checkmark}{=} \sum p_{abs}$



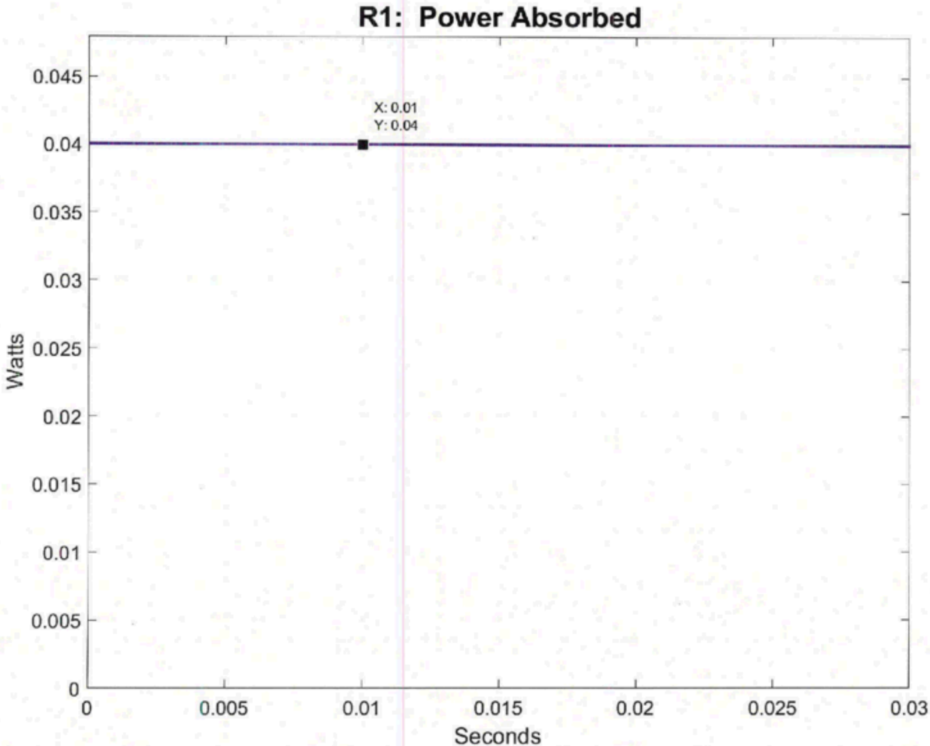
C: Voltage
Volts
Seconds
X: 0.01
Y: 8.647
0
1
2
3
4
5
6
7
8
9
10
0
0.005
0.01
0.015
0.02
0.025
0.03

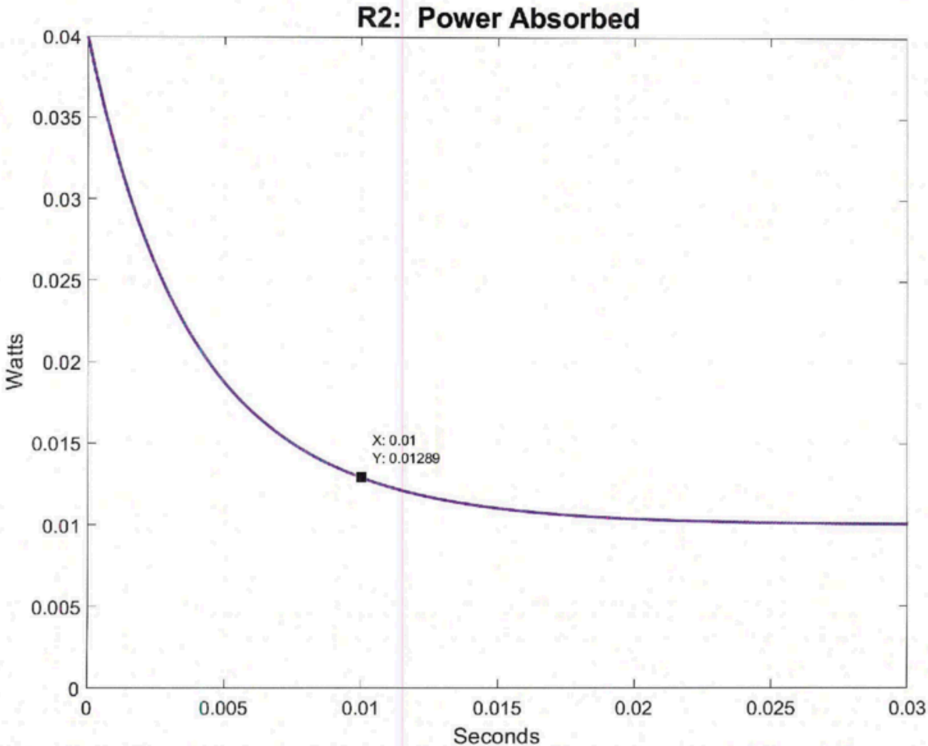

C: Current
×10^-3
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Amperes
X: 0.01
Y: 0.0002706
0
0.005
0.01
0.015
0.02
0.025
0.03
Seconds

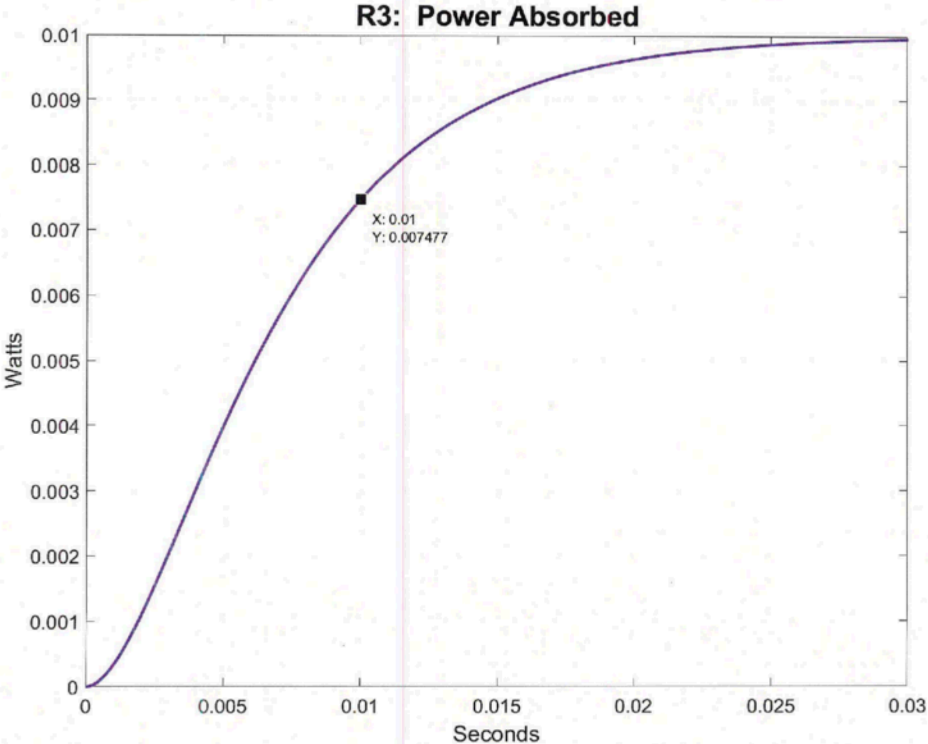

C: Power Absorbed
×10^-3
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
Watts
X: 0.01
Y: 0.00234
0
0.005
0.01
0.015
0.02
0.025
0.03
Seconds

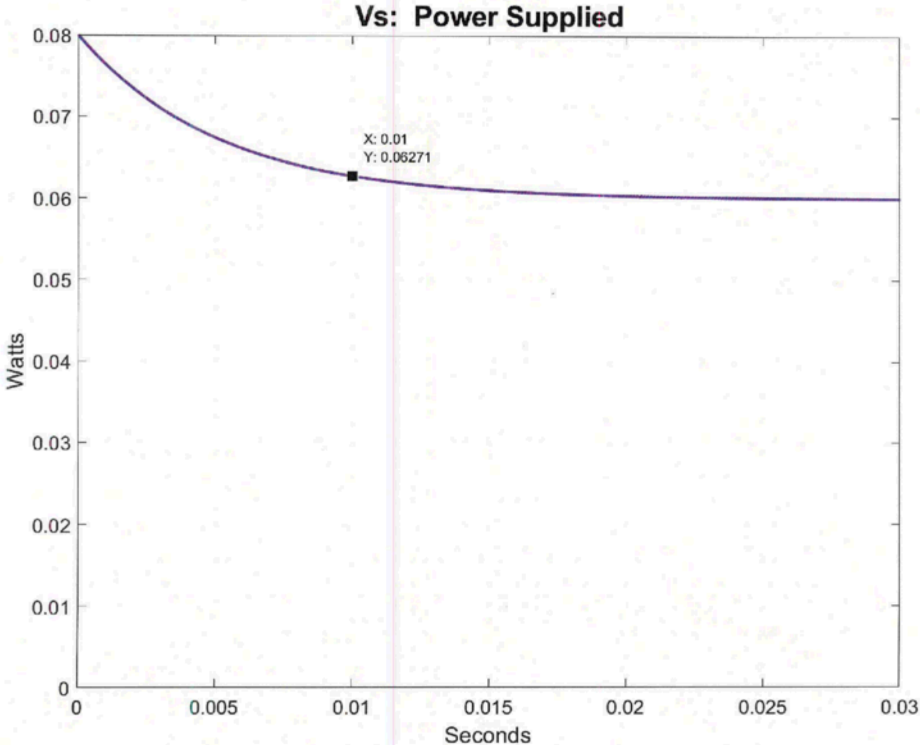


C: Energy Absorbed
×10^-5
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
Joules
X: 0.01
Y: 3.739e-05
0
0.005
0.01
0.015
0.02
0.025
0.03
Seconds

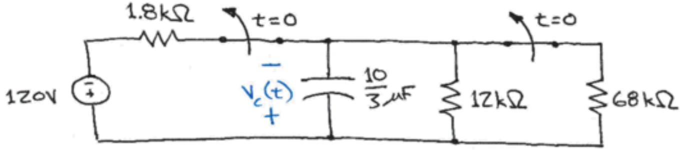

R1: Power Absorbed
X: 0.01
Y: 0.04
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
Watts
0
0.005
0.01
0.015
0.02
0.025
0.03
Seconds


R2: Power Absorbed
Watts
Seconds
X: 0.01
Y: 0.01289
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0.005
0.01
0.015
0.02
0.025
0.03


R3: Power Absorbed
Watts
Seconds
0.01
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
0.005
0.01
0.015
0.02
0.025
0.03
X: 0.01
Y: 0.007477


Vs: Power Supplied
Watts
Seconds
X: 0.01
Y: 0.06271
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
0.005
0.01
0.015
0.02
0.025
0.03

Problem 7.26



First find $v_c(t)$

$$v_c(0) = \frac{10.2}{1.8+10.2}(120) = 102\ \text{V}$$

$$V_{oc} = 0$$

$$\tau = RC = (12\text{k}\Omega)\left(\tfrac{10}{3}\mu\text{F}\right) = 40\text{ms}$$

So, $\underline{v_c(t) = 102e^{-25t}\ \text{V}}$

a) $$P_{12k}(t) = \frac{v_c^2(t)}{12\text{k}\Omega} = 0.867e^{-50t}\ \text{W}$$

$$\omega_{12k}(t) = \int_0^t P_{12k}(\tau)\,d\tau$$

$$= \frac{0.867}{-50}e^{-50\tau}\Big|_0^t$$

$$= 17.34\left(1-e^{-50t}\right)\text{mJ}$$

$\underline{\omega_{12k}(12\text{ms}) = 7.824\ \text{mJ}}$

b) $w_c(t) = \frac{1}{2} C v_c^2(t)$

$$= \frac{1}{2}\left(\frac{10}{3} \times 10^{-6}\right)(102)^2 e^{-50t}$$

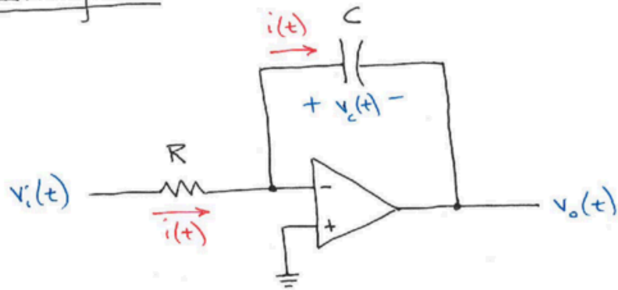
$$= 17.34 e^{-50t} \text{ mJ}$$

$$w_c(t_x) = (.25)(17.34) = 17.34 e^{-50t_x}$$

So, $-50t_x = \ln 0.25$

$$t_x = \frac{\ln 0.25}{-50} = \underline{\underline{27.73 \text{ ms}}}$$

Integrator



$$i(t) = \frac{v_i(t)}{R}$$

$$v_o(t) = -v_c(t) = -\left\{ v_c(0) + \frac{1}{C}\int_0^t i(\tau)\,d\tau \right\}$$

$$= -v_c(0) - \frac{1}{C}\int_0^t \frac{v_i(\tau)}{R}\,d\tau$$

$$= -v_c(0) - \frac{1}{RC}\int_0^t v_i(\tau)\,d\tau$$