STUDY GUIDE: RELATED RATES

AP Classroom Unit 4: Contextual Applications of Differentiation

- 4.1 Videos
  - Remember that the derivative of a function is giving me the slope or rate of change at a particular instant. To determine its units, we can guide ourselves with the known notations. $f'(x) = \frac{df}{dx}$ in $\frac{\text{units of } f}{\text{units of } x}$
  - A derivative is about a particular instant in time, not an interval. When answering a derivative FRQ, remember to talk about the context of $f(x)$, its rate of change, the particular instant, and the units.

- 4.2 Videos
  - Let the function $x(t)$ represent the position a particle has at a particular time t. The first derivative of the function $x(t)$ would be the velocity of the particle with a function $v(t)$. The second derivative of the function $x(t)$ or the first derivative of the function $v(t)$ would be the acceleration of the particle.

$$X(t) \longrightarrow X'(t) = V(t) \longrightarrow X''(t) = V'(t) = a(t)$$

  - Key reminder

When the velocity is negative, the particle is moving to the left.

When the velocity is positive, the particle is moving to the right.

When the velocity and acceleration of the particle have the same signs, the particle's speed is increasing.

When the velocity and acceleration of the particle have opposite signs, the particle's speed is decreasing (or slowing down).

When the velocity is zero and the acceleration is not zero, the particle is momentarily stopped and changing direction.

- If $v(t) = 0$, the object is at rest.
- If $a(t) = 0$, the object could be at rest or moving at a constant speed.

- 4.3 Videos
  - When solving rate of change problems, take into account the following things: a rate of change function helps us predict how a quantity will change over time. State exactly when the quantity is increasing or decreasing and how fast is it changing. Remember derivative notation is not always used.

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- No matter the context, AP tends to ask questions to assess skills such as finding or approximating derivative of a function, interpret the derivative with the right units in context, determine when a quantity is increasing or decreasing.

- Don't forget to justify your answer. Some easy justifications are:

Behavior of $Q(t)$	Evidence using $Q'(t)$
$Q(t)$ is not changing	$Q'(t) = 0$
$Q(t)$ is increasing	$Q'(t) > 0$
$Q(t)$ is decreasing	$Q'(t) < 0$

- Be sure to mention these at some point because that may cost a point in FRQ.

- Rate-in and rate-out problems are infamous for having two rates of change. To find whether a quantity is increasing or decreasing at a specific time or interval, compare the rate in and rate out.

- Easy way to solve these problems. Type into your calculator both rates of change. Create a new function that is the rate in minus the rate out. This new function will tell you whether the overall rate of change is positive or negative. Example question:

The penguin population on an island is modeled by the differentiable function P(t) for $0 \le t \le 40$, where t is in years. There are 100,000 penguins on the island at $t = 0$. The birth rate for the penguins is modeled by $B(t) = 1000e^{0.06t}$ penguins per year and the death rate is modeled by $D(t) = 250e^{0.1t}$ penguins per year.

a) Is the penguin population increasing or decreasing at $t = 5$ years? Give a reason for your answer.

Let $R(t) = B(t) - D(t)$.
$R(5) = 937.678 > 0$

The penguin population is increasing at $t = 5$ years because $R(5) > 0$ which means the birth rate is greater than the death rate at $t = 5$.

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- 4.4 Videos
  - Related rates refers to a function with two or more variables where the independent variable is t. It includes implicit differentiation. Related rates are done with respect to time almost always.

- 4.5 videos
  - Important formulas for related rates:

Shape	Area	Volume	Perimeter	Surface Area	Extras
Square	$A = s^2$		$P = 4s$		
Rectangle	$A = l \cdot w$		$P = 2l + 2w$		
Triangle	$A = \frac{1}{2}bh$		$P = a+b+c$		$c^2 = a^2 + b^2$
Circle	$A = \pi r^2$		$C = 2\pi r$		
Sq. Box		$V = s^3$		$SA = 6s^2$	
Rect. Box		$V = l \cdot w \cdot h$		$SA = 4lw + 2wh$	
Cone		$V = \frac{1}{3}\pi r^2 h$		$SA = \pi r^2 + 2\pi rh$	

- Related rates can be affected by similarity of triangles. Remember to use proportionalities properly.

- Related rates with angles you can do two things: if you have a calculator the find the angle using the inverse of a trig function but if you don't have a calculator you can substitute for the trig ratio. SOH CAH TOA

Function	Derivative
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sin x / \cos^2 x$
$\csc x$	$-\cos x / \sin^2 x$
$\cot x$	$-\csc^2 x$

- The shadow problem:

Practice

What do we know?

$$\frac{dx}{dt} = 4\frac{\text{ft}}{\text{sec}}$$

A person whose height is 6 feet is walking away from the base of a streetlight along a straight path at a rate of 4 feet per second. If the height of the streetlight is 15 feet, what is the rate at which the person's shadow is lengthening?

(A) 1.5 ft/sec (B) 2.667 ft/sec (C) 3.75 ft/sec (D) 6 ft/sec (E) 10 ft/sec

$$\frac{15}{x+z} = \frac{6}{z}$$

$$15z = 6(x+z)$$

$$9z = 6x$$

$$9\frac{dz}{dt} = 6\frac{dx}{dt}$$

$$\frac{dz}{dt} = \frac{2}{3}\frac{dx}{dt} = \frac{2}{3}(4) = \frac{8}{3}\frac{\text{ft}}{\text{sec}}$$