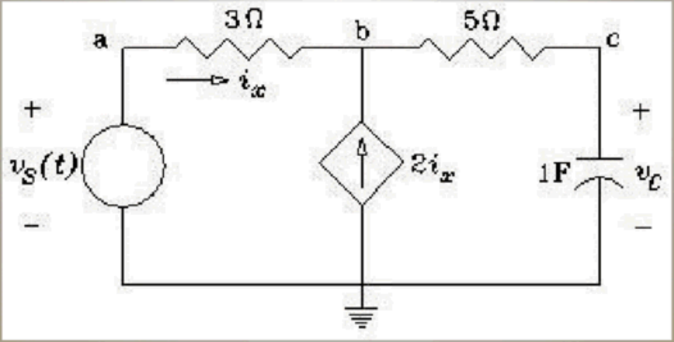
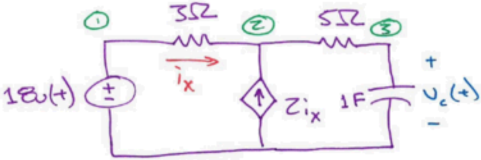
Example 8.7.14 – *RC* Circuit with Dependent Source

- Consider the circuit shown with a controlled current source. The circuit is initially at rest. Find the complete response of the capacitor voltage. The source voltage is $v_S(t) = 18\,u(t)$.

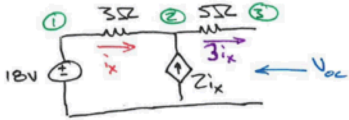




$v_c(0) = 0$

Find $v_c(t)$

• Determine a Thevenin equivalent.



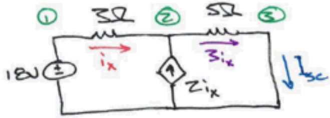
KCL at Node 2:

$$i_x + 2i_x = 3i_x$$

Because of the open circuit:

$$3i_x = 0$$
$$i_x = 0$$

$$V_{oc} = 18\text{V}$$



KCL at Node 2:

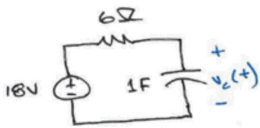
$$i_x + 2i_x = 3i_x$$

KVL around outer loop:

$$-18 + 3i_x + 5(3i_x) = 0$$
$$18i_x = 18$$
$$i_x = 1$$

$$I_{sc} = 3i_x = 3\text{A}$$

$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{18\text{V}}{3\text{A}} = 6\Omega$$

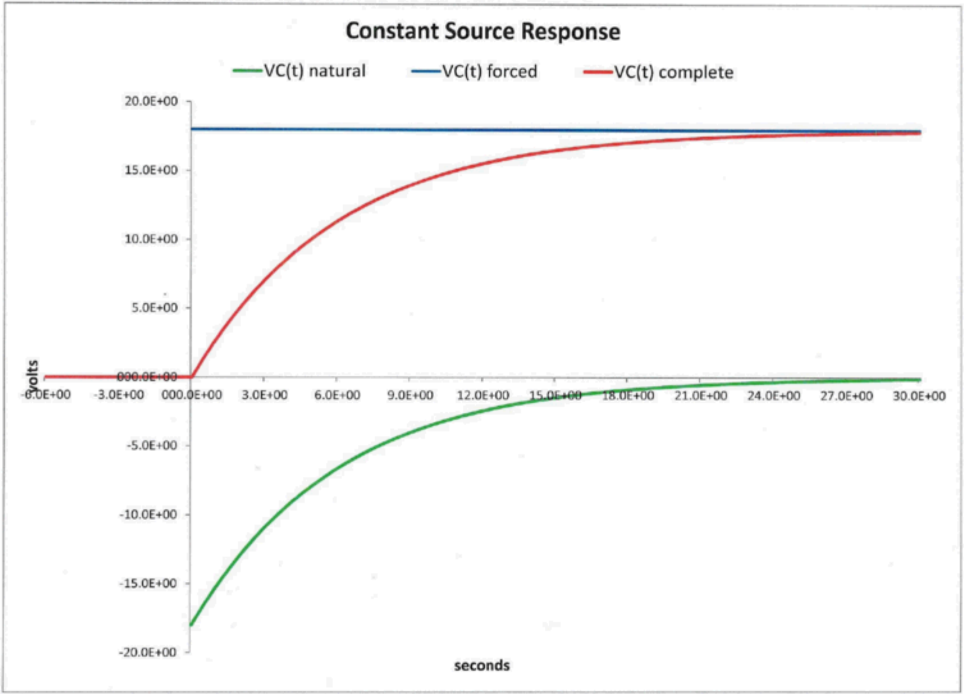


$v_c(0) = 0$

$V_{oc} = 18\text{V}$
$R = 6\Omega$
$C = 1\text{F}$
$\tau = RC = 6 \text{ sec}$

• So,

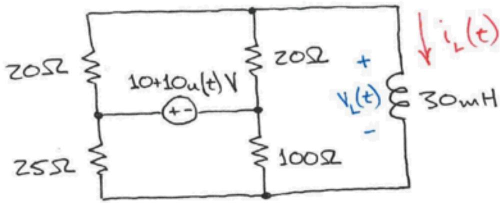
$$v_c(t) = V_{oc} + [v_c(0) - V_{oc}]e^{-t/RC}$$
$$= \underline{\underline{18 - 18e^{-t/6}\text{ V}}}$$



Constant Source Response
VC(t) natural
VC(t) forced
VC(t) complete
20.0E+00
15.0E+00
10.0E+00
5.0E+00
000.0E+00
-5.0E+00
-10.0E+00
-15.0E+00
-20.0E+00
volts
-6.0E+00
-3.0E+00
000.0E+00
3.0E+00
6.0E+00
9.0E+00
12.0E+00
15.0E+00
18.0E+00
21.0E+00
24.0E+00
27.0E+00
30.0E+00
seconds

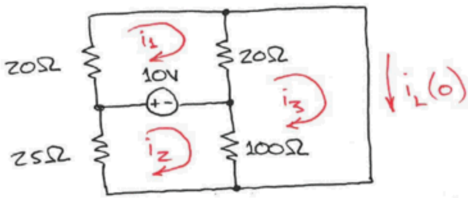
RL Example

Determine $i_L(t)$ and $v_L(t)$.



First find $i_L(0)$.

In the steady-state just prior to $t=0$:



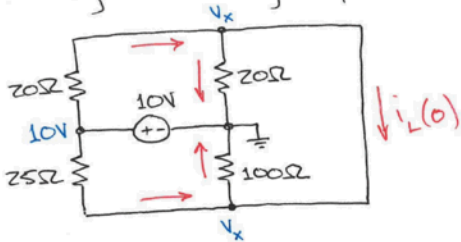
Mesh current equations:

$$20i_1 + 20(i_1 - i_3) - 10 = 0$$
$$25i_2 + 10 + 100(i_2 - i_3) = 0$$
$$20(i_3 - i_1) + 100(i_3 - i_2) = 0$$

$$\begin{bmatrix} 40 & 0 & -20 \\ 0 & 125 & -100 \\ -20 & -100 & 120 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -10 \\ 0 \end{bmatrix}$$

From which $i_3 = i_L(0) = -0.1\text{ A}$

Or using node voltage equations:



$$\frac{10-v_x}{20} = i_L(0) + \frac{v_x}{20}$$

$$\frac{10-v_x}{25} + i_L(0) = \frac{v_x}{100}$$

$\longrightarrow$

$$\frac{10-v_x}{20} - \frac{v_x}{20} = i_L(0)$$

$$\frac{10-v_x}{25} - \frac{v_x}{100} = -i_L(0)$$

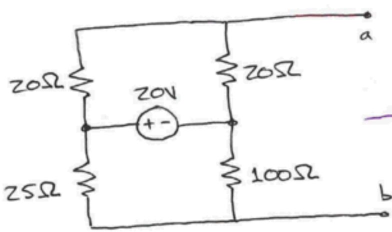
$$\frac{10-v_x}{20} - \frac{v_x}{20} + \frac{10-v_x}{25} - \frac{v_x}{100} = 0$$

From which:

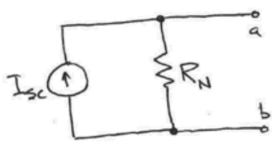
$$v_x = 6V$$

$$i_L(0) = \frac{10-6}{20} - \frac{6}{20} = -0.1A \quad \text{(as before)}$$

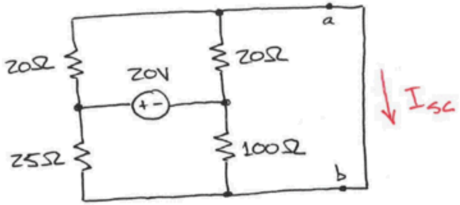
Next find the Norton equivalent for $t \geq 0$



$\longrightarrow$



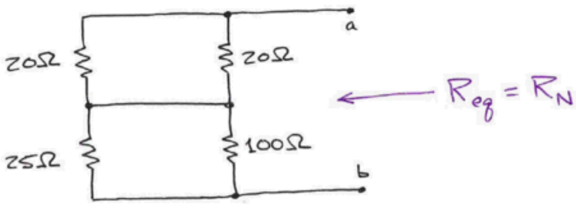
For I_{sc}:



This is the same circuit as the one for finding $i_L(0)$ except the voltage source is doubled (20V vs. 10V). Therefore:

$$I_{sc} = 2\,i_L(0) = -0.2\text{A}$$

For R_N:



$$R_{eq} = 20\Omega \| 20\Omega + 25\Omega \| 100\Omega$$

$$= 10\Omega + 20\Omega$$

$$= 30\Omega$$

So,

$i_L(0) = -0.1\,A$

$I_{sc} = -0.2\,A$

$\tau = \frac{L}{R_N} = \frac{.03}{30} = 1\,ms$

$$i_L(t) = I_{sc} + [i_L(0) - I_{sc}]\, e^{-t/\tau}$$

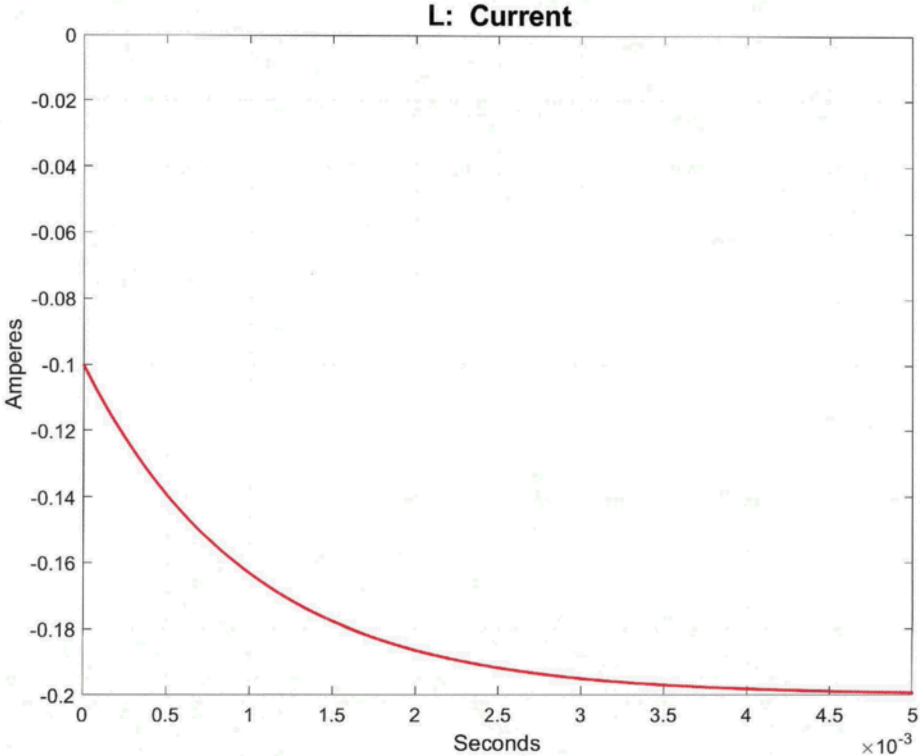
$$= -0.2 + [-0.1 - (-0.2)]\, e^{-t/.001}$$

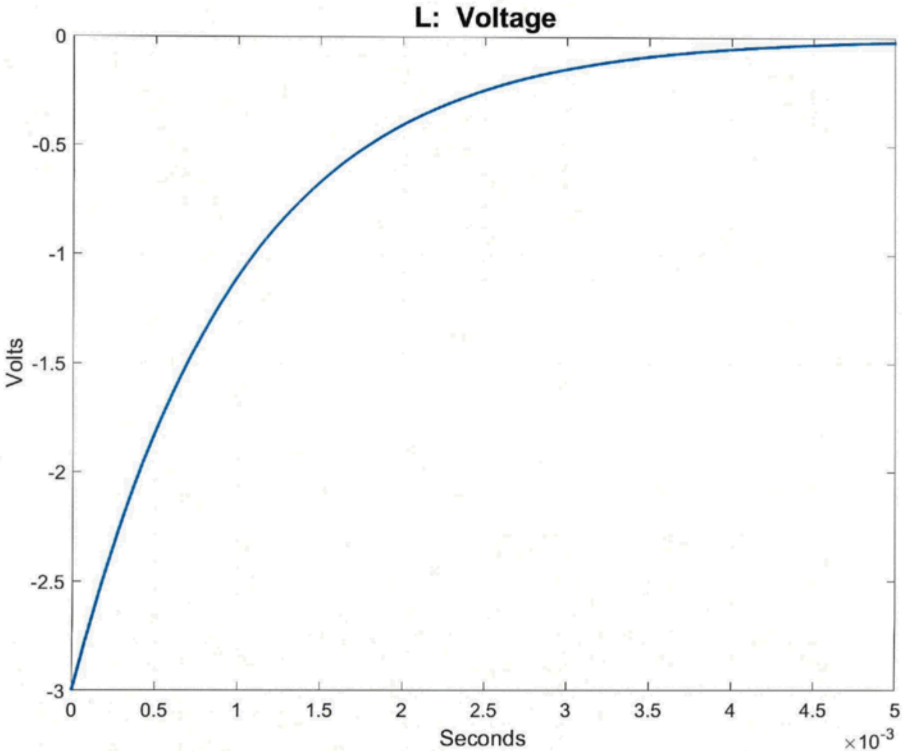
$$= \underline{\underline{-0.2 + 0.1 e^{-1000t}\ A}}$$

$$v_L(t) = L \frac{di_L}{dt}$$

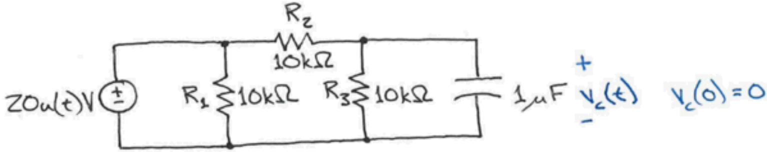
$$= 0.03\,(0.1)(-1000)\, e^{-1000t}$$

$$= \underline{\underline{-3 e^{-1000t}\ V}}$$


L: Current
Amperes
Seconds
×10^-3
0
-0.02
-0.04
-0.06
-0.08
-0.1
-0.12
-0.14
-0.16
-0.18
-0.2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

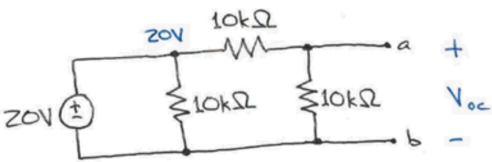

L: Voltage
Volts
Seconds
0
-0.5
-1
-1.5
-2
-2.5
-3
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
×10^-3

Example



- Determine the capacitor voltage, current, power absorbed and stored energy.
- Determine the power absorbed by the resistors.
- Determine the power supplied by the source.
- Evaluate these expressions for $t = 10\,ms$.

First find the Thevenin equivalent circuit facing the capacitor for $t \geq 0$:



By voltage division:

$$V_{oc} = \frac{10}{10+10}(20) = 10V$$