

Suites

1. Suites Numériques

✓ Suites Adjacentes

$(U_n), (V_n)$ adjacentes \Rightarrow convergent et $\lim U_n = \lim V_n$

✓ Relations en 0

$$U_n = O(0) \Leftrightarrow U_n = 0$$

$$U_n = o(0) \Leftrightarrow U_n = 0$$

$$U_n \sim 0 \Leftrightarrow U_n = 0$$

✓ Relations en 1

$$U_n = O(1) \Leftrightarrow (U_n) \text{ bornée}$$

$$U_n = o(1) \Leftrightarrow (U_n) \rightarrow 0$$

$$U_n \sim 1 \Leftrightarrow (U_n) \rightarrow 1$$

✓ D'Alembert

$$\lim \left| \frac{U_{n+1}}{U_n} \right| < 1 \Rightarrow U_n \rightarrow 0$$

$$\lim \left| \frac{U_{n+1}}{U_n} \right| > 1 \Rightarrow U_n \rightarrow +\infty$$

$$\lim \left| \frac{U_{n+1}}{U_n} \right| = 1 \Rightarrow \text{pas de conclusion}$$

✓ Puissances comparées (FEPL)

$$\lim n^\alpha = \begin{cases} +\infty & \text{si } \alpha > 0 \\ 0 & \text{si } \alpha < 0 \end{cases}$$

$$\lim a^n = \lim e^{n \ln(a)} = \begin{cases} +\infty & \text{si } \alpha > 1 \\ 0 & \text{si } 0 < \alpha < 1 \end{cases}$$

✓ Géométrie

$$U_{n+1} = q \cdot U_n$$

$$U_n = q^n \cdot U_0$$

$$S_n = \sum_{k=0}^n q^k = \frac{1 - q^{n+1}}{1 - q}$$

Arithmétique

$$U_{n+1} = U_n + r$$

$$U_n = U_0 + n \cdot r$$

$$S_n = \sum_{k=0}^n k = \frac{n(n+1)}{2}$$

$$T_n = \sum_{k=0}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sqcup_n = \sum_{k=0}^n k^3 = \left(\frac{n(n+1)}{2} \right)^2$$

✓ Arithméco-Géométrie

$$U_{n+1} = a \cdot U_n + b$$

$$l = \frac{b}{1-a}$$

$$V_n = U_n - l$$

$$V_{n+1} = a \cdot V_n \text{ géométrie}$$

✓ Homographique

$$U_{n+1} = \frac{a \cdot U_n + b}{c \cdot U_n + d}$$

$$U_{n+1} = f(U_n)$$

Etude de la fonction

2. Suites de fonctions

$$✓ (f_n) \text{ CU vers } f \text{ sur } I \Leftrightarrow \lim_n \|f_n - f\|_\infty^l = 0$$

✓ Continuité

$$\left. \begin{array}{l} \forall n, f_n \text{ cont en } a \\ (f_n) \text{ CU} \end{array} \right\} \Rightarrow f \text{ cont en } a$$

✓ Double limite

$$(f_n) \text{ CU} \Rightarrow \lim_{x \rightarrow a} \lim_n f_n(x) = \lim_n \lim_{x \rightarrow a} f_n(x)$$

$$\mathbf{CU \Rightarrow CS \Rightarrow CA}$$