

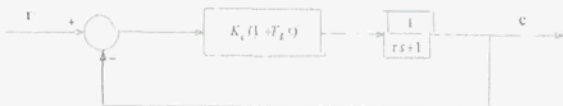
Closed book/notes. Adequate work must be shown to receive credit for problem. Mark answers. There are eight pages, 100 points and 25 bonus points for this exam. A formula sheet is provided which includes some data for water and Laplace Transform theory.

(Pts)

- (6) 1 Discuss/describe the basic operating principle of each of the following control modes. Indicate which have offset and which have no offset.

- a). Proportional - controller output is proportional to the error (difference between actual value of the controlled variable and the desired value or set point) has offset
- b). PD - controller output is proportional to the error and the derivative of the error with respect to time has offset
- c). PID - controller output is proportional to the error, the time integral of the error and the derivative with respect to time of the error. I mode eliminates offset

- (8) 2. a. Determine  $\frac{C}{R}(s)$  for the feedback control system shown below. Put the transfer function in standard form.



$$\frac{C}{R}(s) = \frac{G}{1 + GH} = \frac{\frac{N}{D}}{1 + \frac{N}{D}(s)} = \frac{N}{D + N} = \frac{K_c(1 + T_d s)}{\tau s + 1 + K_c(1 + T_d s)}$$

$$= \frac{K_c(T_d s + 1)}{(\tau + K_c T_d)s + K_c + 1}$$

$$\frac{C}{R}(s) = \frac{\frac{K_c}{1 + K_c}(T_d s + 1)}{\left(\frac{\tau + K_c T_d}{1 + K_c}\right)s + 1}$$

$$\text{OR } G_{cl} = \frac{K_c(1 + T_d s)}{\tau s + 1}$$

$$\frac{C}{R}(s) = \frac{\frac{K_c(1 + T_d s)}{\tau s + 1}}{1 + \frac{K_c(1 + T_d s)}{\tau s + 1}(1)}$$

then manipulate to form

$$\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sX(s)$$

- (4) b. Determine whether this control system has offset by using either the final value theorem in Laplace Transforms or the parameters of the system transfer function.

$$\lim_{t \rightarrow \infty} \frac{C}{R}(t) = \lim_{s \rightarrow 0} s \frac{\frac{K_c}{1+K_c} (T_d s + 1)}{\frac{1 + K_c T_d}{1 + K_c} s + 1} \frac{1}{s} = \frac{K_c}{1+K_c} < 1$$

∴ have offset

OR gain of  $\frac{C}{R}(s) = \frac{K_c}{1+K_c}$  which is less than 1 ∴ have offset.

- (4) 3. Describe the basic operating principles of a negative feedback control system.

A process variable for which it is desired to maintain a particular value is measured and the current value compared to the desired value or setpoint. If there is a difference a signal that is a function of this difference is sent to a control device (e.g. valve, cylinder) that controls an input to the process and returns the controlled variable to desired state.

- (4) 4. What are the two main uses or functions of a feedback control system?

- to maintain the controlled variable at the desired value in the face of disturbances
- to change the controlled variable in a prescribed manner.

5. In tuning a PD controller using the ultimate or Zeigler-Nichols method, the ultimate gain was determined to be 2.5 gpm/psi and the ultimate period as 1.4 minutes.

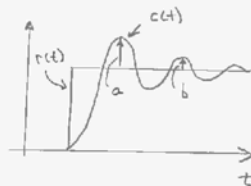
- (6) a. What would be the recommended gains for operating this system?

$$K_c = \frac{1.5}{2.5} \frac{\text{gpm}}{\text{psi}} = 0.6 \left( \frac{2.5 \text{ gpm}}{\text{psi}} \right)$$

$$T_d = \frac{0.175}{8} \text{ min} = \frac{1.4 \text{ minute}}{8}$$

- (4) b. Relative to a step change in the setpoint of this system, what objective does this method of tuning a control system attempt to meet? Show a sketch to support your discussion.

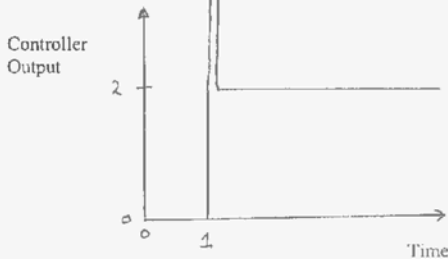
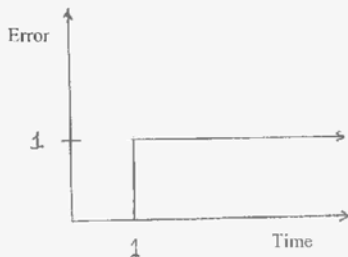
$\frac{1}{4}$  decay ratio - the ratio of the second overshoot to the first overshoot =  $\frac{1}{4} = \frac{b}{a}$



- (4) 6. What are the two major concerns or challenges when employing the derivative control mode in feedback control system?

- taking the derivative tends to amplify any noise present
- derivative control mode not effective when error is constant (derivative = zero).

- (6) 7. A PD controller experiences a change in the error signal as shown on the graph on the left. Sketch the output of the controller on the graph to the right noting maximum and steady state values. Error and controller output are both in volts and  $K_c = 2$  and  $T_d = 3$ . Controller transfer function:  $K_c (1 + T_d s)$



- (3) 8. a. What is the theoretical minimum sampling speed required to digitize a signal? What happens when the sampling speed is too slow?

theoretical minimum is 2 times the highest frequency present

too slow causes aliasing - the development of data (frequencies) not actually in the signal.

- (1) b. What is the recommended sampling speed to accurately capture amplitude data?

~ 8 to 10 times the highest frequency present,

$$* \max W_A = [(25 \text{ m})(0.0027 \frac{\text{lb}_f}{\text{m}^3}) + (0.18 \frac{\text{lb}_f}{\text{m}^3})(0.2 \text{ m})] = 0.0675 + 0.036 = 0.1035 \text{ psi}$$

- (9) 9. A U-tube manometer is to be used to measure the pressure (psi) in a vessel. The specific weight of the manometer fluid is  $0.18 \text{ lb}_f/\text{in}^3 \pm 1.5\%$  and the difference in the height of the fluid in the manometer (h) can be determined within  $\pm 0.2 \text{ in}$ . Determine the pressure (psig) in the vessel and its root sum square uncertainty if  $h = 25 \text{ inches}$ .

$$P = \gamma h \quad \frac{4.50 \frac{\text{lb}_f}{\text{m}^2}}{0.18 \frac{\text{lb}_f}{\text{in}^3}} = 0.18 \frac{\text{lb}_f}{\text{in}^3} (25 \text{ m}) \quad \frac{\partial P}{\partial h} = \gamma \quad \frac{\partial P}{\partial \gamma} = h$$

$$W_\gamma = \frac{0.0027 \frac{\text{lb}_f}{\text{m}^3}}{0.18 \frac{\text{lb}_f}{\text{in}^3}} = 0.015 (0.18 \frac{\text{lb}_f}{\text{in}^3}) \quad W_h = 0.2 \text{ m}$$

$$W_R = \left[ \left( \frac{\partial P}{\partial \gamma} W_\gamma \right)^2 + \left( \frac{\partial P}{\partial h} W_h \right)^2 \right]^{\frac{1}{2}} = \left\{ \left[ (25 \text{ m})(0.0027 \frac{\text{lb}_f}{\text{m}^3}) \right]^2 + \left[ (0.18 \frac{\text{lb}_f}{\text{m}^3})(0.2 \text{ m}) \right]^2 \right\}^{\frac{1}{2}}$$

$$= (0.004556 + 0.001216)^{\frac{1}{2}} = 0.0765 \text{ psi}$$

$$P = 4.50 \pm 0.0765 \text{ psi} \quad (1.7\%)$$

$$\text{OR} \quad \frac{W_P}{P} = \left[ \sum \left( \frac{a_i w_i}{X_i} \right)^2 \right]^{\frac{1}{2}} = \left\{ \left[ \frac{(1)(0.0027 \frac{\text{lb}_f}{\text{m}^3})}{0.18 \frac{\text{lb}_f}{\text{m}^3}} \right]^2 + \left[ \frac{(1)(0.2 \text{ m})}{25 \text{ m}} \right]^2 \right\}^{\frac{1}{2}}$$

$$= [(0.015)^2 + (0.008)^2]^{\frac{1}{2}} = 0.017 \quad (1.7\%)$$

$$W_P = \frac{0.0765 \text{ psi}}{4.50 \text{ psi}} = 0.017 (4.5 \text{ psi})$$

- (5) 10. What would be the maximum range of the pressure (psi) measured in the previous problem if the errors were combined in the most detrimental way?

$$P = \gamma h \quad \gamma = 0.985 (0.18 \frac{\text{lb}_f}{\text{m}^3}) + 0.1015 (0.18 \frac{\text{lb}_f}{\text{m}^3})$$

$$\Rightarrow 0.1773 \frac{\text{lb}_f}{\text{m}^3} \leq \gamma \leq 0.1827 \frac{\text{lb}_f}{\text{m}^3}$$

$$h \Rightarrow 25 \text{ m} - 0.2 \text{ m} < h < 25 \text{ m} + 0.2 \text{ m} \Rightarrow 24.8 \text{ m} \leq h \leq 25.2 \text{ m}$$

$$\max P = \frac{4.604 \frac{\text{lb}_f}{\text{m}^2}}{0.1773 \frac{\text{lb}_f}{\text{m}^3}} = 0.1827 \frac{\text{lb}_f}{\text{m}^3} (25.2 \text{ m})$$

$$\min P = \frac{4.397 \frac{\text{lb}_f}{\text{m}^2}}{0.1827 \frac{\text{lb}_f}{\text{m}^3}} = 0.1773 \frac{\text{lb}_f}{\text{m}^3} (24.8 \text{ m})$$

$$\text{Range} = 0.207 \text{ psi} = 4.604 - 4.397$$

$$P = 4.50 \pm 0.1035 \text{ psi} = 4.50 \pm \frac{0.207 \text{ psi}}{2}$$

$$\text{OR} \quad \frac{W_P}{P} = \frac{(1)(0.0027 \frac{\text{lb}_f}{\text{m}^3})}{0.18 \frac{\text{lb}_f}{\text{m}^3}} + \frac{(1)(0.2 \text{ m})}{25 \text{ m}} = 0.015 + 0.008$$

$$= 0.023$$

$$W_P = \frac{0.1035 \text{ psi}}{4.50 \text{ psi}} = 0.023 (4.50 \text{ psi})$$

Note: Improving  $\gamma$  would have most impact on improving  $P$ .

third method using  
general expression  
 $W_R$  at top of page \*

11. You have been assigned to design a data acquisition system that will collect pressure data from a hydraulic system in a modern tractor with its higher pressure ranges. The pressure transducer available has a diaphragm that deflects under pressure and is instrumented with four strain gages all active and connected in a Wheatstone bridge. A data acquisition card interfaces the transducer with the computer for data collection.

Known Parameters:

- Working pressure range of transducer: 0 - 3000 psi
- The output from the Wheatstone bridge goes from 0 to 30 mV over the working range of the transducer.
- The output of the bridge is conditioned by a differential op-amp to utilize the full range of the ADC on the data acquisition card
- The data acquisition card has an analog input range of 0 to 10 volts and the ADC resolution is 10 bits.

- (3) a). What is the smallest change in pressure (psi) that your system will be able to detect?

$$\text{Res} = \frac{2.93 \text{ psi}}{2^{10}} = \frac{3000 \text{ psi}}{2^{10}} = \frac{3000 \text{ psi}}{1024}$$

- (3) b). Write an equation relating the pressure,  $P$ , to voltage output from the signal conditioning

$$P - \text{psi} = \frac{3000 \text{ psi}}{10 \text{ Volts}} (\text{Volts}) \Rightarrow \boxed{P = 300 \times \text{Volts}}$$

- (6) c). Write an equation relating pressure to the ADC counts value.

$$P = \frac{1}{2} (2.93 \text{ psi}) + \frac{2.93 \text{ psi}}{\text{count}} \times (\text{counts}) \pm \frac{2.93 \text{ psi}}{2}$$

$$\text{or } \boxed{P = \frac{1}{2} \frac{3000 \text{ psi}}{2^{10}} + \frac{3000 \text{ psi}}{2^{10}} (\text{counts}) \pm \frac{3000 \text{ psi}}{2^{11}}}$$

- (6) d). After acquiring an analog reading the register holding the ADC result reads <sup>9816543210</sup>1100101101<sub>2</sub>. What is the pressure (psi)?

$$\boxed{813 \text{ counts}} = 1 \times 2^9 + 1 \times 2^8 + 1 \times 2^5 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^0$$

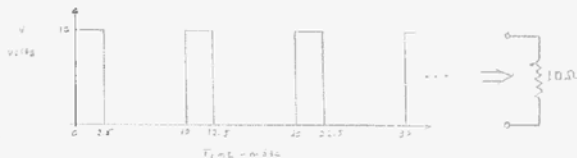
$$\boxed{P = \frac{2387.3 \text{ psi}}{2} \pm 1.46 \text{ psi}} = \frac{1}{2} \frac{3000 \text{ psi}}{2^{10}} + \frac{3000 \text{ psi}}{2^{10}} (813 \text{ counts}) \pm \frac{3000 \text{ psi}}{2^{11}}$$

- (3) e). Which parameter would you change, and to what value to be able to sense a 0.1 psi change in pressure?

change number of bits from 10 to 16 bits

$$0.046 \text{ psi} = \frac{3000 \text{ psi}}{2^{16}} \quad 156.75 \text{ mV} (0.092 \text{ psi})$$

12. The voltage across a  $10\Omega$  resistor as a function of time is shown in the graph below.



- (5) a.. What is the RMS voltage across the resistor?

$$V_{\text{Rms}} = \sqrt{\frac{1}{T} \int_0^T V^2 dt}$$

$$\text{Integral} = 0.25 \text{ V}^2 \text{ sec} = (10 \text{ V})^2 (0.0025 \text{ sec})$$

$$V^2 \text{ avg} = \frac{25}{0.01 \text{ sec}} \text{ V}^2 = \frac{0.25 \text{ V}^2 \text{ sec}}{0.01 \text{ sec}}$$

$$V_{\text{Rms}} = \frac{5.0}{\text{V}} = \sqrt{25 \text{ V}^2}$$

- (2) b. Using the results in part a what is the average power (W) being converted to heat in the resistor?

$$\frac{2.50 \text{ W}}{\text{W}} = \frac{(5 \text{ V}_{\text{Rms}})^2}{10 \Omega}$$

- (2) c.. What is the duty cycle (%) of the voltage signal?

$$\frac{25}{100} \% = \frac{2.5 \text{ mSec}}{10 \text{ mSec}} \times 100$$

- (2) d. What is the average power (W) dissipated by the resistor if the voltage is continuous at 10V?

$$\frac{10}{\text{W}} = \frac{(10 \text{ V})^2}{10 \Omega}$$

- (2) e. What is the ratio (%) of the power at the duty cycle shown to that with a duty cycle of 100% and how does this compare to the duty cycle?

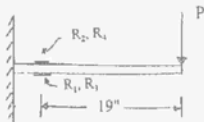
$$\frac{25}{100} \% = \frac{2.5 \text{ W}}{10 \text{ W}} \times 100 \quad \text{Same}$$

- (2) f. What is the frequency (Hz) of the voltage signal?

$$\frac{100}{\text{Hz}} = \frac{1 \text{ cycle}}{0.01 \text{ sec}} \quad \frac{1 \text{ Hz-sec}}{\text{cycle}}$$

**Bonus**

- (8) 13. Four  $350\Omega$  strain gages are attached to a steel beam as shown below. If  $P = 30$  lbs,  $E = 30 \times 10^6$  psi and the gage factor is 2.07 what would be the expected change in the resistance ( $\Omega$ ) of one of the gages?



Beam Cross Section



$$I = \frac{6953 \times 10^{-3}}{12} \text{ in}^4 = \frac{1}{12} (1.5 \text{ in}) (0.25 \text{ in})^3$$

$$\sigma = \frac{Mc}{I} \quad c = \frac{0.25 \text{ in}}{2} = \frac{0.25 \text{ in}}{2}$$

$$M = \frac{570}{12} \text{ in-lb} = (19 \text{ in}) (30 \text{ lb})$$

$$\sigma = \frac{36,450}{\text{in}^2} = \frac{(570 \text{ in-lb}) (0.125 \text{ in})}{1.953 \times 10^{-3} \text{ in}^4}$$

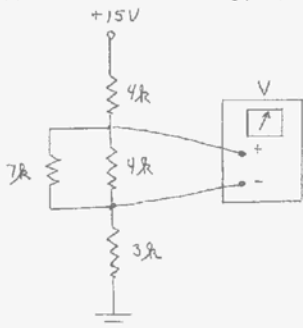
$$E = \frac{\sigma}{\epsilon} \Rightarrow \epsilon = \frac{\sigma}{E}$$

$$\epsilon = \frac{1216}{30 \times 10^6} \text{ in/in} = \frac{36,450 \text{ psi}}{30 \times 10^6 \text{ psi}} \times \frac{10^6 \mu\epsilon}{\epsilon}$$

$$\Delta R = GF E \epsilon$$

$$\Delta R = \frac{0.88}{10^6} \text{ ohms} = 2.07 (1216 \mu\epsilon) (350 \Omega) \times \frac{\epsilon}{10^6 \mu\epsilon}$$

- (4) 14. What would be the reading (volts) on the voltmeter shown below. The voltmeter is ideal.



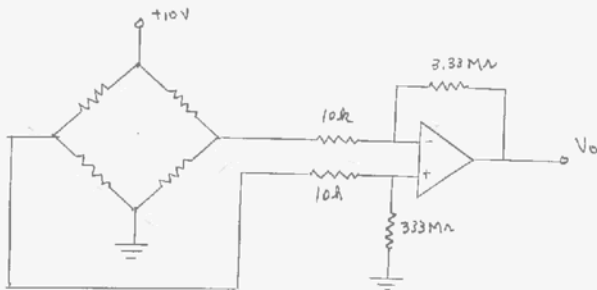
$$R_{eq} = \frac{2.545}{1} \text{ k} = \frac{(7 \text{ k})(4 \text{ k})}{(7+4) \text{ k}}$$

$$V = \frac{4.0}{4 \text{ k} + 2.545 \text{ k} + 3 \text{ k}} (15 \text{ V})$$

- (7) 15. Design and show the schematic (with values for all resistors) for a circuit to interface to the load cell in problem #11 to the ADC. The signal coming from the circuit should make use of the full range of the ADC.

$$\text{Amplification} = \underline{333} = \frac{10\text{V}}{30\text{mV}} \times \frac{1000\text{mV}}{V}$$

$$\text{Gain} = \frac{R_1}{R_2} = 333 \quad \text{if } R_2 = 10\text{k}\Omega \quad \text{then } R_1 = 333(10\text{k}\Omega) = 3.33\text{M}\Omega$$



- (3) 16. What is the specific gravity of the manometer fluid in problem #9

$$\boxed{S_p = \frac{4.98}{1.03} = 0.18 \frac{\text{lbf}}{\text{in}^3} \frac{5+3}{62.4 \text{lbf}} \frac{(728 \text{in}^3)}{5+3}}$$

- (3) 17. What is the value of  $A6D_{16}$  in base 10?

$$\boxed{2669} = 10 \times 16^2 + 6 \times 16^1 + 13 \times 16^0$$

0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	A
11	B
12	C
13	D
14	E
15	F



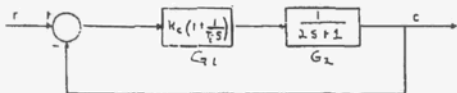
Closed book/notes. Adequate work must be shown to receive credit for problem. Mark answers. There are seven pages, 115 points and 15 bonus points for this exam. A formula sheet is provided which includes some data for water and pressure conversions and Laplace Transform theory.

(Pts)

- (6) 1. Discuss/describe the basic operating principle of each of the following control modes. Indicate which have offset and which have no offset.

- a). Proportional - controller output is proportional to the error (difference between actual value of the controlled variable and the desired value or set point) has offset
- b). PI - controller output is proportional to the error and the integral of the error with respect to time, mode eliminates offset
- c). PID controller output is proportional to the error, the time integral of the error and the derivative with respect to time of the error, mode eliminates offset

- (8) 2. a. Determine  $\frac{C}{R}(s)$  for the feedback control system shown below. Put the transfer function in standard form.



$$\text{Controller } G_1(s) = K_c + \frac{K_c}{T_i s} = \frac{K_c T_i s + K_c}{T_i s}$$

$$G = G_1 G_2 = \frac{K_c T_i s + K_c}{T_i s (2s + 1)} \quad H: H(s) = 1.0$$

$$\frac{C}{R}(s) = \frac{G}{1 + GH} = \frac{\frac{K_c T_i s + K_c}{T_i s (2s + 1)}}{1 + \frac{K_c T_i s + K_c}{T_i s (2s + 1)}}$$

$$= \frac{K_c T_i s + K_c}{2 T_i s^2 + T_i s + K_c T_i s + K_c} = \frac{K_c T_i s + K_c}{2 T_i s^2 + (1 + K_c T_i) T_i s + K_c}$$

$$\boxed{\frac{C}{R}(s) = \frac{T_i s + 1}{\frac{2 T_i}{K_c} s^2 + \frac{1 + K_c T_i}{K_c} s + 1}}$$

$$\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} s X(s)$$

- (4) b. Determine whether the control system in problem #2 has offset by using either the final value theorem in Laplace Transforms.

$$\begin{aligned} \lim_{t \rightarrow \infty} \frac{c}{R}(t) &= \lim_{s \rightarrow 0} s \frac{T_c s + 1}{\frac{3T_c}{K_b} s^2 + \frac{1 + K_c T_c}{K_c} s + 1} \cdot \frac{1}{s} \\ &= 1 \Rightarrow \text{no offset} \end{aligned}$$

- (4) 3. Describe the basic operating principles of a negative feedback control system. A variable for which it is desired to maintain at a particular value is measured and the current value compared to the desired value or setpoint. If there is a difference a signal that is a function of this difference is sent to a control device (e.g. valve) that controls an input to the process and returns the controlled
- (4) 4. What are the two main uses or functions of a feedback control system? variable & desired state.

- to maintain the controlled variable at the desired value in the face of disturbances
- to change the controlled variable in a prescribed manner.

5. In tuning a PI controller using the ultimate or Zeigler-Nichols method, the ultimate gain was determined to be 2 gpm/°F and the ultimate period as 1.2 minutes.

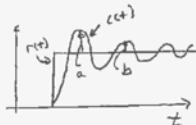
- (6) a). What would be the recommended gains for operating this system?

$$K_c = \frac{0.9}{2} \frac{\text{gpm}}{\text{°F}} = 0.45 \left( \frac{2 \text{ gpm}}{\text{°F}} \right) = 0.45 \text{ Su}$$

$$T_i = \frac{1.0}{1.2} \text{ min} = \frac{1.2 \text{ min}}{1.2} = \frac{P_u}{1.2}$$

- (4) b). Relative to a step change in the setpoint of this system, what objective does this method of tuning a control system attempt to meet? Show a sketch to support your discussion.

$\frac{1}{4}$  decay ratio - the ratio of the second overshoot to first overshoot =  $\frac{1}{4} = \frac{b}{a}$



- (2) 6. What are the positive and negative results of increasing the controller gains in negative feedback control system.

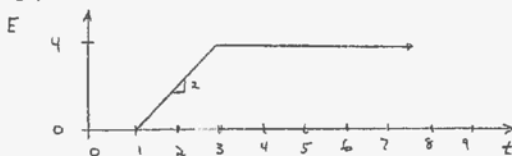
Positive: faster response and reduced offset

Negative: decreases stability

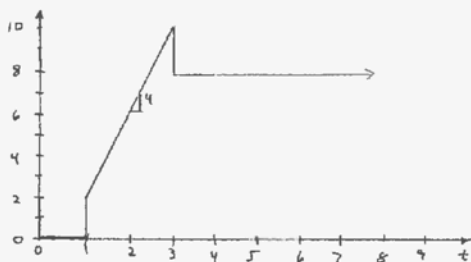
- (4) 7. What are the two major concerns or challenges when employing the derivative control mode in feedback control system?

- taking derivative of signal tends to amplify any noise present
- derivative control mode not effective when error is constant (derivative = zero)

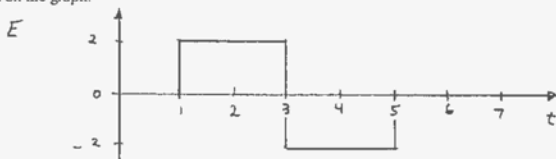
- (4) 8. A PD controller experiences a change in the error signal as shown in the first graph. Sketch the output of the controller on the second graph.  $K_c = 2$  and  $T_D = 1$ . Make sure the values of any slopes and horizontal lines are noted on the graph.



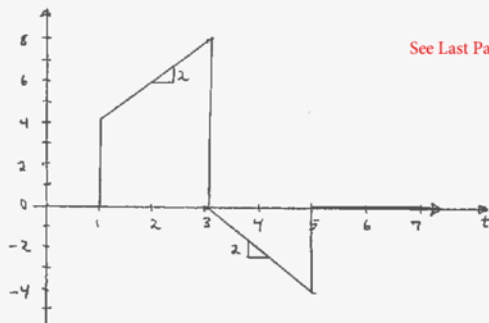
PD Controller output



- (4) 9. A PI controller experiences a change in the error signal as shown in the first graph. Sketch the output of the controller on the second graph.  $K_c = 2$  and  $T_I = 1$ . Make sure the values of any slopes and horizontal lines are noted on the graph.



PI Controller Output



See Last Page

(9) 10.. Define each of the following properties and give a typical set of English and SI units.

a). Density mass per unit volume  $\frac{\text{lbm}}{\text{ft}^3}$ ,  $\frac{\text{kg}}{\text{m}^3}$

b). Specific weight weight per unit volume  $\frac{\text{lb}_f}{\text{ft}^3}$ ,  $\frac{\text{N}}{\text{m}^3}$

c). Specific gravity ratio of the density or specific weight of a substance to the density or specific weight of water  
dimensionless

(6) 11. The level of liquid product in a 10 ft diameter holding tank (top vented to the atmosphere) for a process line could be sensed using a pressure gage at the bottom of the tank. If this gage reads 10 psi and the specific gravity of the product is 0.89 what is the height (ft) of the product in the tank?

$$p = \text{SG} \times \gamma_{H_2O} h \Rightarrow h = \frac{p}{\text{SG} \times \gamma_{H_2O}}$$

$$h = \frac{25.9}{\text{ft}} = \frac{10 \frac{\text{lb}_f}{\text{in}^2} \times \frac{\text{ft}^2}{0.89 (62.4 \frac{\text{lb}_f}{\text{ft}^3})}}{\frac{144 \text{in}^2}{\text{ft}^2}}$$

(4) 12. The intake pressure to a centrifugal pump is 5 in Hg vac and the discharge pressure is 15 psig. What pressure drop (psi) across the pump?

$$\Delta p = \frac{12.5}{\text{psi}} = 15 \text{ psig} - 5 \text{ in Hg} \times \frac{1 \text{ psi}}{2.04 \text{ in Hg}}$$

(3) 13. What is the absolute pressure (psia) at the intake of the pump in problem #12 if the local atmospheric pressure is 30.1 in Hg ABS?

$$P_{\text{ABS}} = P_{\text{atm}} + P_{\text{gage}}$$

$$p = \frac{12.3}{\text{psia}} = [30.1 \text{ in Hg ABS} - 5 \text{ in Hg VAC}] \frac{1 \text{ psi}}{2.04 \text{ in Hg}}$$

(3) 14. a. What is the theoretical minimum sampling speed required to digitize a signal? What happens if the sampling speed is too slow?

2 times the highest frequency present

too slow causes aliasing - the development of data (frequencies) not actually in the signal

b. What is the recommended sampling speed to accurately capture amplitude data?

~ 8 to 10 times the highest frequency present

- (8) 15. The power being dissipated in a 5% resistor with a nominal value of  $12\Omega$  is to be calculated from the current through it. If this current is measured as  $I = 8 \pm 0.15A$  calculate the power (watts) in the resistor and the root sum square uncertainty (watts) in the power.  $P = I^2 R$

$$\frac{\partial P}{\partial I} = 2IR, \quad \frac{\partial P}{\partial R} = I^2 \quad w_R = \frac{0.6}{12} = 0.05 (12\Omega)$$

$$\frac{\partial P}{\partial I} = 2(8A)(12\Omega) = 192 \frac{W}{A}, \quad \frac{\partial P}{\partial R} = (8A)^2 = 64 \frac{W}{\Omega}$$

$$w_P = \left\{ \left[ \frac{\partial P}{\partial I} w_I \right]^2 + \left[ \frac{\partial P}{\partial R} w_R \right]^2 \right\}^{\frac{1}{2}} = \left\{ \left[ 192 \frac{W}{A} (0.15A) \right]^2 + \left[ 64 \frac{W}{\Omega} (0.6\Omega) \right]^2 \right\}^{\frac{1}{2}}$$

$$= \left[ (28.8W)^2 + (38.4W)^2 \right]^{\frac{1}{2}} = 48W$$

$$P_{\text{nominal}} = 768W = (8A)^2 (12\Omega)$$

$$\Rightarrow \boxed{P = 768W \pm 48W}$$

$$\text{OR} \quad \frac{w_P}{P} = \left[ \sum \left( \frac{a_i w_i}{x_i} \right)^2 \right]^{\frac{1}{2}} = \left[ \left( \frac{2(0.15A)}{8A} \right)^2 + \left( \frac{1(0.6\Omega)}{12\Omega} \right)^2 \right]^{\frac{1}{2}} = 0.0625$$

$$w_P = 48W = 0.0625 (768W)$$

$$\boxed{P = 768W \pm 48W}$$

- (8) 16. What would be the maximum range of the power (watts) measured in the previous problem if the errors were combined in the most detrimental way? Determine using both methods.

a). Method one  $P_{\text{max}} = 836.9W = (8.15A)^2 (12.6\Omega)$

$$P_{\text{min}} = 702.5W = (8 - 0.15A)^2 (12 - 0.6\Omega)$$

$$\text{Range} = 134.4W = 836.9W - 702.5W$$

$$P = 768W \pm \frac{134.4W}{2}$$

$$\Rightarrow \boxed{P = 768W \pm 67.2W}$$

- b). Method two

$$w_{P_{\text{max}}} = 67.2W = 28.8W + 38.4W = \frac{\partial P}{\partial I} w_I + \frac{\partial P}{\partial R} w_R$$

$$\boxed{P = 768W \pm 67.2W}$$

- 17 You have been assigned to design a data acquisition system that will collect pressure data from a hydraulic system. The pressure transducer available has a diaphragm that deflects under pressure and is instrumented with four strain gages all active and connected in a Wheatstone bridge. A data acquisition card interfaces the transducer with the computer for data collection.

Known Parameters:

1. Working pressure range of transducer: 0 - 2000 psi
2. The output from the Wheatstone bridge goes from 0 to 30 mV over the working range of the transducer.
3. The output of the bridge is conditioned by a differential op-amp to utilize the full range of the ADC on the data acquisition card
4. The data acquisition card has an analog input range of 0 to 5 volts and the ADC resolution is 12 bits.

- (3) a). What is the gain (V/V) on the differential op-amp?

$$\boxed{\frac{167}{1} \frac{V}{V}} = \frac{5V}{30mV} \times \frac{1000mV}{V}$$

- (3) b). What is the smallest change in pressure (psi) will your system be able to recognize?

$$\boxed{\frac{0.489}{1} \text{ psi}} = \frac{2000 \text{ psi}}{2^{12}} = \frac{2000 \text{ psi}}{4096}$$

- (3) c). Write an equation relating the pressure, P, to voltage output from the signal conditioning

$$P = \frac{2000 \text{ psi}}{5V} \times V_0$$

$$\boxed{P = \frac{400 \text{ psi}}{V} \times V_0}$$

- (6) d). Write an equation relating pressure to the ADC counts value.

$$P = \frac{1}{2} \frac{2000 \text{ psi}}{4096} + \frac{2000 \text{ psi}}{4096} (\text{counts}) \pm \frac{1}{2} \frac{2000 \text{ psi}}{4096}$$

$$\boxed{P = 0.244 + 0.488 (\text{counts}) \pm 0.244 \text{ psi}}$$

- (6) e). After acquiring an analog reading the register holding the ADC result reads 110101101. What is the pressure (psi)?  $2^6 + 2^5 + 2^3 + 2^2 + 2^0 = 429$

$$\text{--- psi} = \frac{1}{2} \frac{2000 \text{ psi}}{4096} + \frac{2000 \text{ psi}}{4096} (429 \text{ counts}) \pm \frac{1}{2} \frac{2000 \text{ psi}}{4096}$$

$$\boxed{\text{--- psi} = 209.7 \pm 0.244 \text{ psi}}$$

- (3) f). Which parameter would you change to what value to be able to sense a 0.1 psi change in pressure?

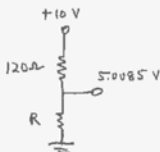
$$\text{increase } n \text{ to } 16 \text{ bit} \quad 2^{16} = 65,536$$

$$Res = \frac{0.031}{1} \text{ psi} = \frac{2000 \text{ psi}}{2^{16}} < 0.1 \text{ psi} \Rightarrow \text{OK}$$

$$\frac{0.1 \text{ psi}}{2^{15}} = \frac{2000 \text{ psi}}{2^{15}} < 0.1 \text{ psi} \\ \text{OK } n = 15$$

**Bonus**

- (15) 18. A  $120\Omega$  strain gage is being used to determine the stress in a steel structural member. It is connected into a Wheatstone bridge which has three precision  $120\Omega$  resistors in addition to the strain gage and a 10 volt excitation voltage. (The strain gage is at position  $R_1$  of the Wheatstone bridge listed as item #6 on the formula sheet.) If the output of the bridge is 8.5 mV, the gage factor of the strain gage is 2.07 and the modulus of elasticity of steel is  $30 \times 10^6$  psi, what is the stress (psi) in the structural member? Is the member in tension or compression?



$$\frac{R}{R+120} (10V) = 5.0085V$$

$$R = 0.50085 (R+120)$$

$$(1 - 0.50085) R = 0.50085 (120)$$

$$R = \frac{120.4087\Omega}{1 - 0.50085} = \frac{(0.50085)(120)}{1 - 0.50085}$$

$$\Delta R = 0.4087\Omega \sim 120.4087\Omega - 120\Omega$$

$$\Delta R = GF \epsilon R \Rightarrow \epsilon = \frac{\Delta R}{GF R}$$

$$1645.3 \mu\epsilon = \frac{0.4087\Omega}{(2.07)(120\Omega)} \times \frac{10^6 \mu\epsilon}{\epsilon}$$

$$\epsilon = \frac{\sigma}{E} \rightarrow \sigma = E \epsilon$$

$$\sigma = \frac{49,359 \text{ lbf}}{\text{in}^2} = \left( 30 \times 10^6 \frac{\text{lbf}}{\text{in}^2} \right) (1645.3 \mu\epsilon) \times \frac{1 \epsilon}{10^6 \mu\epsilon}$$

Tension

$$R = \frac{FL}{A} \quad R \uparrow \text{ when } L \uparrow \text{ (and } A \uparrow) \quad R \uparrow \rightarrow L \uparrow \therefore \text{ tension}$$

# Problem 9)

