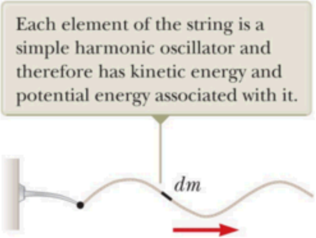
Power of a Wave



$$K_\lambda = \frac{1}{4}\mu\omega^2 A^2 \lambda \qquad U_\lambda = \frac{1}{4}\mu\omega^2 A^2 \lambda$$

$$E_\lambda = U_\lambda + K_\lambda = \frac{1}{2}\mu\omega^2 A^2 \lambda$$

$$P = \frac{T_{\mathrm{MW}}}{\Delta t} = \frac{E_\lambda}{T} = \frac{\frac{1}{2}\mu\omega^2 A^2 \lambda}{T} = \frac{1}{2}\mu\omega^2 A^2 \left(\frac{\lambda}{T}\right)$$

$$\boxed{P = \frac{1}{2}\mu\omega^2 A^2 v}$$

Week 14 HW Q2

A sinusoidal wave on a string is described by the wave function

$$y = 0.10 \sin(0.85x - 39t)$$

where x and y are in meters and t is in seconds. The mass per unit length of this string is 12.0 g/m.

(a) Determine the speed of the wave.

45.9 m/s

(b) Determine the wavelength.

7.39 m

(c) Determine the frequency.

6.21 Hz

(d) Determine the power transmitted by

4.19 W

Comparing the given wave function

$$y = (0.21)\sin(0.87x - 46t)$$

with the general form of the wave function

$$y = A\sin(kx - \omega t),$$

we find that

$$k = 0.87 \text{ rad/m}$$

$$\omega = 46 \text{ rad/s}$$

$$A = 0.21 \text{ m.}$$

(a) The wave speed is given by

$$v = f\lambda = \frac{\omega}{k} = \frac{46 \text{ rad/s}}{0.87 \text{ rad/m}} = 52.9 \text{ m/s.}$$

(b) The wavelength is given by

$$\lambda = \frac{2\pi}{k} = \frac{2\pi \text{ rad}}{0.87 \text{ rad/m}} = 7.22 \text{ m.}$$

(c) The frequency is given by

$$f = \frac{\omega}{2\pi} = \frac{46 \text{ rad/s}}{2\pi \text{ rad}} = 7.32 \text{ Hz.}$$

(d) The power of the wave is given by the following equation.

$$P = \frac{1}{2}\mu\omega^2A^2v = \frac{1}{2}(0.012 \text{ kg/m})(46 \text{ s}^{-1})^2(0.21 \text{ m})^2(52.9 \text{ m/s}) = 29.6 \text{ W.}$$

(a) We will need to apply the Doppler equation twice. First, let's calculate the frequency of the chirp as observed by the mosquito. Using our Doppler equation, $f = f_{\text{chirp}}$, $f' = f_{\text{observed by mosquito}}$, and because the bat and mosquito are moving in the same direction, we have $v_S = +v_{\text{bat}}$ and $v_O = -v_{\text{mosquito}}$. Applying our equation, we get

$$f' = f\left(\frac{v + v_O}{v - v_S}\right) \to f_{\text{observed by mosquito}} = f_{\text{chirp}}\left(\frac{v - v_{\text{mosquito}}}{v - v_{\text{bat}}}\right).$$

Now, the sound of the chirp is reflected off the mosquito, so we can use $f_{\text{observed by mosquito}}$ as f, the frequency of the source. The observer is now the bat, which is moving towards the source (the mosquito), so $v_O = +v_{\text{bat}}$, the source is now the mosquito, moving away from the observer, so $v_S = -v_{\text{mosquito}}$. Finally, $f' = f_{\text{echo}}$. Applying the Doppler equation, we get

$$f' = f\left(\frac{v + v_O}{v - v_S}\right) \to f_{\text{echo}} = f_{\text{observed by mosquito}}\left(\frac{v + v_{\text{bat}}}{v - (-v_{\text{mosquito}})}\right).$$

Putting our two equations together, we get

$$f_{\text{echo}} = f_{\text{chirp}}\left(\frac{v - v_{\text{mosquito}}}{v - v_{\text{bat}}}\right)\left(\frac{v + v_{\text{bat}}}{v + v_{\text{mosquito}}}\right).$$

We need to solve this for v_{mosquito}.

$$\frac{f_{\text{echo}}}{f_{\text{chirp}}}\frac{(v - v_{\text{bat}})}{(v + v_{\text{bat}})} = \frac{v - v_{\text{mosquito}}}{v + v_{\text{mosquito}}}$$

Let's do a substitution to make our calculation cleaner.

Let

$$M = \frac{f_{\text{echo}}}{f_{\text{chirp}}}\frac{(v - v_{\text{bat}})}{(v + v_{\text{bat}})}.$$

So we have

$$M = \frac{v - v_{\text{mosquito}}}{v + v_{\text{mosquito}}}.$$

Solving for v_{mosquito}, we get

$$Mv + Mv_{\text{mosquito}} = v - v_{\text{mosquito}}$$

$$Mv_{\text{mosquito}} + v_{\text{mosquito}} = v - Mv$$

$$v_{\text{mosquito}}(1 + M) = v(1 - M)$$

Let's go ahead and solve for M.

$$M = \frac{f_{\text{echo}}}{f_{\text{chirp}}}\frac{(v - v_{\text{bat}})}{(v + v_{\text{bat}})} = 0.9908$$

Finally, we can find v_{mosquito}.

$$v_{\text{mosquito}} = (343\text{ m/s})\frac{(1 - 0.9908)}{(1 + 0.9908)} = 1.59\text{ m/s}$$

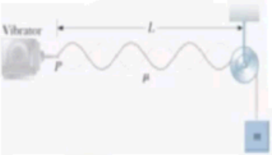
(b) The bat will catch the mosquito if its speed is greater than the mosquito's speed, or $v_S > v_O$.

In this case we have

$$4.36\text{ m/s} > 1.59\text{ m/s},$$

so the bat does catch the mosquito.

In the arrangement shown in the figure, an object can be hung from a string (with linear mass density $\mu = 0.002\ 00$ kg/m) that passes over a light pulley. The string is connected to a vibrator (of constant frequency f), and the length of the string between point P and the pulley is L = 2.00 m. When the mass m of the object is either 16.0 kg or 25.0 kg, standing waves are observed; no standing waves are observed with any mass between these values, however. What is the frequency of the vibrator? Note: the greater the tension in the string, the smaller the number of nodes in the standing wave.



$n \to 25$

$n+1 \to 16$

$\lambda = \frac{2L}{n}$, $f = \frac{v}{\lambda}$

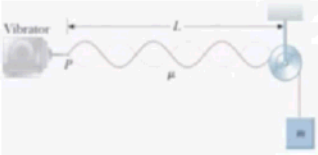
$$f = \frac{n}{2L}\sqrt{\frac{T_n}{\mu}} , \quad f = \frac{n+1}{2L}\sqrt{\frac{T_{n+1}}{\mu}} \Rightarrow \frac{n+1}{n} = \sqrt{\frac{T_n}{T_{n+1}}} = \sqrt{\frac{(25.0kg)g}{(16.0kg)g}} = \frac{5}{4}$$

$4n + 4 = 5n$

$n = 4$

$$\Rightarrow f = \frac{4}{2(2.00m)}\sqrt{\frac{(25.0kg)(9.80m/s^2)}{0.00200\ kg/m}} = 350.$$

In the arrangement shown in the figure, an object can be hung from a string (with linear mass density $\mu = 0.002\ 00$ kg/m) that passes over a light pulley. The string is connected to a vibrator (of constant frequency f), and the length of the string between point P and the pulley is L = 2.00 m. When the mass m of the object is either 16.0 kg or 25.0 kg, standing waves are observed; no standing waves are observed with any mass between these values, however. What is the largest object mass for which standing waves could be observed?



$T_n = mg$

$$f = \frac{n}{2L}\sqrt{\frac{T_n}{\mu}} = \frac{n}{2L}\sqrt{\frac{mg}{\mu}}$$

$$\Rightarrow m = \frac{4L^2 f^2 \mu}{n^2 g}$$

$n = 1$

$$m = \frac{4(2.00m)^2(350Hz)^2(0.00200kg/m)}{(1)^2(9.80m/s^2)} = 400!$$