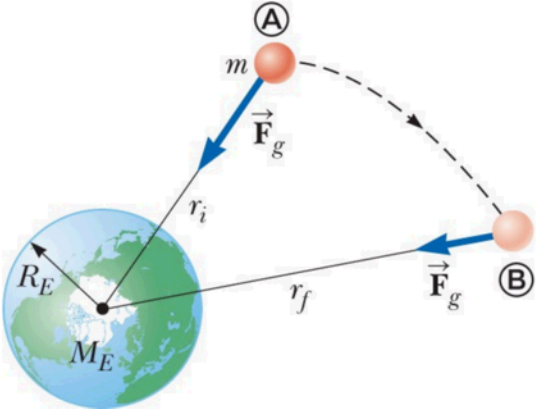
Energy Considerations in Planetary and Satellite Motion

$$E = K + U_g \Rightarrow E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

$$\Delta K + \Delta U_g = 0 \Rightarrow E_i = E_f$$



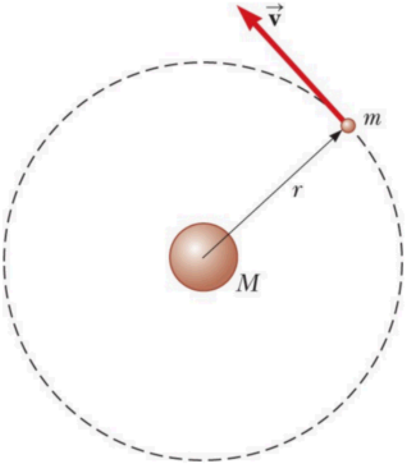
Energy Considerations in Planetary and Satellite Motion

$$F_g = ma \Rightarrow \frac{GMm}{r^2} = \frac{mv^2}{r} \qquad \frac{1}{2}mv^2 = \frac{GMm}{2r}$$

$$E = \frac{GMm}{2r} - \frac{GMm}{r}$$

$$E = -\frac{GMm}{2r} \quad \text{(circular orbits)}$$

$$\boxed{E = -\frac{GMm}{2a} \quad \text{(elliptical orbits)}}$$



Week 12 HW Q3

A 967-kg satellite orbits the Earth at a constant altitude of 105-km.

(a) How much energy must be added to the system to move the satellite into a circular orbit with altitude 192 km?

395 MJ

(b) What is the change in the system's kinetic energy?

-395 MJ

(c) What is the change in the system's potential energy?

790 MJ

A 1000-kiilogram satellite orbits the Earth at a constant altitude of 100 kilometers. How much energy must be added to the system to move the satellite into a circular orbit with altitude 200 kilometers? On analyzing this problem, we see that the total energy needed is equal to the negative of the gravitational constant times the mass of the Earth times the mass of the satellite divided by 2 times the distance between them. So the change in energy is equal to the gravitational constant times the mass of the Earth times the mass of the satellite divided by 2, 1 over the initial orbital radius - 1 over the final orbital radius. This is equal to 6.67 times 10 to the negative 11th newton meters squared per kilogram squared the gravitational constant times the mass of the Earth 5.98 times 10 to the 24th meters divided by 2 times 10 to the third kilograms the mass of the satellite divided by 10 to the third meters all times 1 divided by 6370 + 100 - 1 divided by 6370 + 200. So the change in energy is equal to 4.69 times 10 to the eighth joules, which is equal to 469 megajoules.

>A 1000-kilogram satellite orbits the Earth at a constant altitude of 100 kilometers. What are the changes in the system's kinetic energy when it moves to an altitude of 200 kilometers? On analyzing this problem, we see that both in the original orbit and in the final orbit the total energy is negative. The absolute value of the total energy = the kinetic energy. The potential energy is also negative, and it is equal to twice the total energy. As the satellite is lifted from the lower to the higher orbit, the gravitational energy increases, the kinetic energy decreases, and the total energy increases. Numerically, the gravitational energy increases by 938 megajoules. The kinetic energy decreases by 469 megajoules. And the total energy increases by 469 megajoules.

A 1 000-kg satellite orbits the Earth at a constant altitude of 100 km. How much energy must be added to the system to move the satellite into a circular orbit with altitude 200 km?

$$E_{tot} = -\frac{GMm}{2r}$$

$$\Delta E = \frac{GMm}{2}\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

$$= \frac{(6.67\times10^{-11}\,N\text{-}m^2/kg^2)(5.98\times10^{24}\,m)}{2} \frac{10^3\,kg}{10^3\,m}\left(\frac{1}{6370+100} - \frac{1}{6370+200}\right)$$

$$\Delta E = 4.69\times10^{8}\,J = 469\,MJ$$

What is the change in the system's kinetic/ potential energy?

E_{tot} is neg. $|E_{tot}| = K$

U is neg. $U = 2E_{tot}$

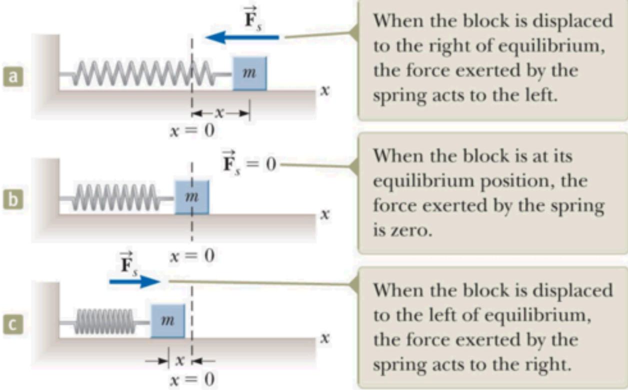
U inc. by 938 MJ change in potential

K dec. by 469 MJ change in kinetic *negative*

E_{tot} inc. by 469 MJ

Ch. 15: Motion of an Object Attached to a Spring

$$F_s = -kx$$



$$\sum F_x = ma_x \Rightarrow -kx = ma_x \Rightarrow a_x = -\frac{k}{m}x$$

Particle in Simple Harmonic Motion

$$a_x = -\frac{k}{m}x \quad \Rightarrow \frac{d^2x}{dt^2} = -\frac{k}{m}x \qquad \omega^2 = \frac{k}{m}$$

$$\boxed{\frac{d^2x}{dt^2} = -\omega^2 x}$$

$$\boxed{x(t) = A\cos(\omega t + \phi)}$$

$$\boxed{v = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi)} \qquad v_{\max} = \omega A = \sqrt{\frac{k}{m}}A$$

$$\boxed{a = \frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \phi)} \qquad a_{\max} = \omega^2 A = \frac{k}{m}A$$

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Week 13 HW Q1

$$x(t) = A\cos(\omega t + \phi)$$

A 0.420-kg object attached to a spring with a force constant of 8.00 N/m vibrates in simple harmonic motion with an amplitude of 12.2 cm. (Assum the position of the object is at the origin at $t = 0$.)

(a) Calculate the maximum value of its speed.

53.2 cm/s

(b) Calculate the maximum value of its acceleration.

232 cm/s^2

(c) Calculate the value of its speed when the object is 10.20 cm from the equilibrium position.

29.2 cm/s

(d) Calculate the value of its acceleration when the object is 10.20 cm from the equilibrium position.

-194 cm/s^2

(e) Calculate the time interval required for the object to move from $x = 0$ to $x = 4.20$ cm.

0.0805 s

Part 3 of 7 - Analyze

The angular frequency is

$$\omega = \sqrt{\frac{k}{m}}$$

$$= \sqrt{\frac{8 \text{ N/m}}{0.47 \text{ kg}}}$$

$$= 4.13 \text{ s}^{-1}.$$

Remember the implied radians in the units of this answer!

Part 4 of 7 - Analyze

Assuming the position of the object is at the origin at $t = 0$ position is given by

$$x = (13.1 \text{ cm}) \sin\left[(4.13 \text{ s}^{-1})t\right],$$

where x is in cm.

(a) The extreme values of the velocity v are given by $\pm\ \omega A$, so we have

$$v_{max} = A\omega$$

$$= (13.1 \text{ cm})(4.13 \text{ s}^{-1})$$

$$= 54.1 \text{ cm/s}.$$

(b) The extreme values of the acceleration a are given by $\pm A\omega^2$, so we have

$$a_{max} = A\omega^2$$

$$= (13.1 \text{ cm})(4.13 \text{ rad/s})^2$$

$$= 223 \text{ cm/s}^2.$$

Part 5 of 7 - Analyze

(c) We solve for t in the equation giving position as a function of time to find

$$t = \frac{1}{4.13 \text{ s}^{-1}} \sin^{-1}\left(\frac{x}{13.1 \text{ cm}}\right).$$

When $x = 10.10$ cm, we have

$$t = \left[\frac{1}{4.13 \text{ rad}} \sin^{-1}(0.771)\right] \text{s}$$

$$= 0.213 \text{ s}.$$

Part 6 of 7 - Analyze

Now that we know t at $x = 10.10$ cm, we find for the speed

$$v = \frac{dx}{dt}$$

$$= (54.1 \text{ cm/s}) \cos\left[(4.13 \text{ s}^{-1})(0.213 \text{ s})\right]$$

$$= 34.5 \text{ cm/s}.$$

(d) The acceleration is

$$a = \frac{dv}{dt}$$

$$= -(223 \text{ cm/s}^2) \sin\left[(4.13 \text{ s}^{-1})(0.213 \text{ s})\right]$$

$$= -172 \text{ cm/s}^2.$$

Part 7 of 7 - Analyze

(e) Using the equation we found above for t, we have

$$t = \frac{1}{4.13 \text{ s}^{-1}} \sin^{-1}\left(\frac{x}{13.1 \text{ cm}}\right)$$

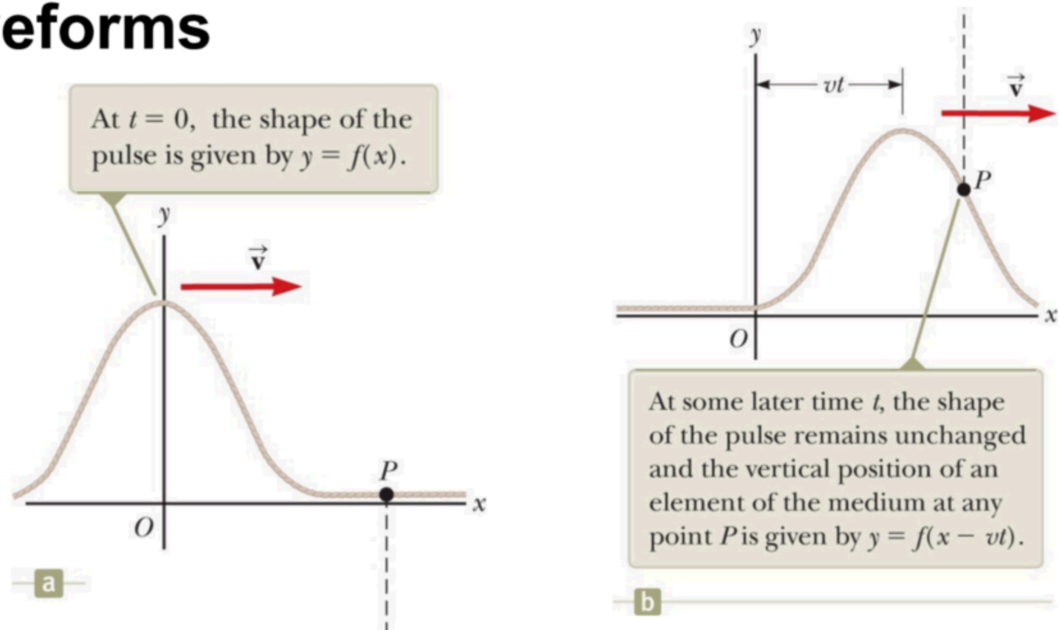
When $x = 0$, $t = 0$, and when $x = 4.10$ cm, we find that

$$t = 0.0771 \text{ s}.$$

Therefore, the time interval

$$\Delta t = 0.0771 \text{ s}.$$

Ch 16: Wave Functions and Waveforms



Mathematical Descriptions of Waves

$$y(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x - vt)\right] \qquad \Rightarrow y(x,t) = A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$$

$$v = \frac{\Delta x}{\Delta t} = \frac{\lambda}{T}$$

$$\boxed{k \equiv \frac{2\pi}{\lambda}} \text{ and } \boxed{\omega \equiv \frac{2\pi}{T} = 2\pi f}$$

$$\boxed{y(x,t) = A\sin(kx - \omega t)}$$

$$v = \frac{\omega}{k} \text{ and } \boxed{v = \lambda f}$$

$$y(x,t) = A\sin(kx - \omega t + \phi)$$