

AP Precalculus Things to Memorize by Unit (roughly)  
 You can use this to help you come up with your "Brain Dump" assignments

Unit 1: Polynomial and Rational Functions & Unit 2: Exponential and Logarithmic Functions

Shapes of functions (parent or in general):

Linear:  
(positive)



Quadratic:  
(positive)



Cubic:  
(positive)



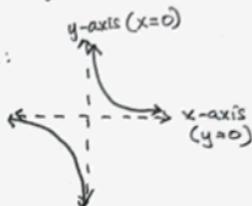
Quartic:  
(positive)



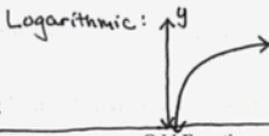
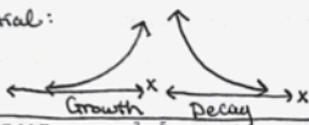
Square Root:  
(positive)



Rational:  
(parent)



Exponential:



Odd Degree:  $x, x^3, x^5$ , etc.

$$f(x) = x^3 + 2x^2 - x - 1$$

vs. Odd Function: rotational symmetry ( $180^\circ$ )

$$\text{Rule: } f(-x) = -f(x)$$

$$\begin{aligned} f(-3) &= 6 \\ f(3) &= -6 \end{aligned}$$

Even Degree:  $x^2, x^4, x^6$

$$g(x) = x^2 + 2x - 4$$

vs. Even Function: y-axis symmetry

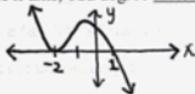
$$\text{Rule: } f(-x) = f(x)$$

$$\begin{aligned} f(-3) &= 6 \\ f(3) &= 6 \end{aligned}$$

Multiplicity: even degree factors bounce off the x-axis, odd degree factors go through the x-axis

Example:  $f(x) = (x-1)(x+2)^2$

$$\begin{array}{l|l} x-1=0 & x+2=0 \\ \hline x=1 & x=-2 \end{array}$$



Transformations (ALL functions,  $f$ ):  $af(b(x-c)) + d$

- Vertical dilation by a factor of  $|a|$ . If  $a < 0$ , then there is a reflection over the x-axis (vertical).
- Horizontal dilation by a factor of  $|\frac{1}{b}|$ . If  $b < 0$ , then there is a reflection over the y-axis (horizontal).
- Horizontal translation  $c$  units.  $(x-3)$  shifts right 3 units and  $(x+3)$  shifts left 3 units
- Vertical translation  $d$  units.

Rates of Change:

$f(x)$  is increasing at an increasing rate = the graph is rising and concave up  $\curvearrowright$

$f(x)$  is increasing at a decreasing rate = the graph is rising and concave down  $\curvearrowleft$

$f(x)$  is decreasing at an increasing rate = the graph is falling and concave up  $\curvearrowleft$

$f(x)$  is decreasing at a decreasing rate = the graph is falling and concave down  $\curvearrowright$

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### Rational Functions (how to find features):

Horizontal Asymptotes: Use the 3 rules about the degrees of the numerator and denominator

Degree of numerator < Degree of denominator: HA is  $y = 0$

Degree of numerator = Degree of denominator: HA is  $y =$  the leading coefficients

Degree of numerator > Degree of denominator: HA is undefined (may have a slant asymptote)

Vertical Asymptotes and Holes:

Factor the function.

Set the denominator = 0 and solve.

If the factor appears in the numerator and denominator, there is a hole. (removable discontinuity)

If the factor only appears in the denominator, there is a VA. (non-removable discontinuity)

Intercepts:

x-intercept  $(a, 0)$ : Set numerator = 0 (after "canceling"/removing factors).

y-intercept  $(0, b)$ : Substitute in  $x = 0$ .

### Inverse functions:

Notation for inverse:  $f^{-1}(a) = b \rightarrow f(b) = a$

Example:  $f^{-1}(2)$ , then  $f(x) = 2$

The x and y values swap, so the domain and range swap.

Invertible function (one-to-one) – when a function's inverse is also a function.

There are unique outputs for every unique input.

### Composite functions:

Notation for composite:  $(f \circ g)(x) = f(g(x))$

Example: For  $f(g(2))$  and  $g(2) = -1$ , then  $f(-1) = f(g(2))$

You are substituting one function/function's output into another.

**Know your Exponent rules and Logarithmic rules:** Examples below

$$6^{2x+1} = (6^2)^x \cdot 6^1 = (36)^x \cdot 6 = 6 \cdot (36)^x \quad \text{and} \quad \log_b b = 1, \log_b 1 = 0$$

$$3^{-1} = \frac{1}{3}, \text{ and } 25^{(1/2)} = \sqrt{25} = 5 \quad \text{and} \quad \log_3 x + 2 \log_3 y - \log_3 z = \log_3 \left( \frac{xy^2}{z} \right)$$

**Solving:** Either find alike bases or isolate and rewrite

$$4^{3x} = 8^{1-2x} \quad \text{and} \quad \log_2(x+2) \cdot x = 3$$

$$2^{2(3x)} = 2^{3(1-2x)} \quad \text{and} \quad 2^3 = x^2 + 2x \quad \text{*you have to know how to factor}$$

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Free Response Question reminders

**Justifying models for FRQ #1:**

Question B (possibly)

Linear:

The input values change at a constant rate. The output values change at a constant rate.

OR The input values change additively. The output values change additively.

Exponential:

The input values change at a constant rate. The output values change proportionally.

OR The input values change at a additively. The output values change multiplicatively.

Logarithmic:

The input values change proportionally. The output values change at a constant rate.

OR The input values change multiplicatively. The output values change additively.

Quadratic:

The input values change at a constant rate. The output values change at a constant 2<sup>nd</sup> difference.

Cubic:

The input values change at a constant rate. The output values change at a constant 3<sup>rd</sup> difference.

**Limit notation:**

Question B (possibly):  $\lim_{x \rightarrow -\infty} f(x) = \infty$  (left end behavior),  $\lim_{x \rightarrow \infty} f(x) = \infty$  (right end behavior)

**Justifying if a function is invertible:**

Question C (possibly):

The function does not have an inverse because the output values for not unique to each input value.

For example,  $f(3) = 2$  and  $f(7) = 2$  \*This would be found on a table or graph given in the problem.

**FRQ #2:**

For question A(i) write two or three equations using the two points provided  $(x_1, y_1)$ ,  $(x_2, y_2)$  with the model equation provided.  $f(x_1) = y_1$  and  $f(x_2) = y_2$

For question A(ii) use the calculator to solve for constants  $a$  and  $b$  (possibly  $c$ ).

For question B(i) "Average Rate of Change" means to use the slope formula:

For question B(iii) you need to talk about how the linear model you found is "secant" to the given model in question A.