1.5 → Integração Linha Elástica ($\overline{BC}$)

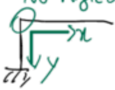
$$y''_{BC} = -\frac{M}{EI} = -\frac{1}{EI}\left(-7{,}5x^2 + 60x - 120\right) = \frac{1}{EI}\left(7{,}5x^2 - 60x + 120\right)$$

$$y'_{BC} = \frac{1}{EI}\left(7{,}5\frac{x^3}{3} - 60\frac{x^2}{2} + 120x + C_3\right)$$

$$y_{BC} = \frac{1}{EI}\left(7{,}5\frac{x^4}{12} - 60\frac{x^3}{6} + 120\frac{x^2}{2} + C_3 \cdot x + C_4\right)$$

1.6 → Condições de Fronteira

Nó Rígido



$$\Rightarrow \varphi_B^{esq} = \varphi_B^{dir} \Rightarrow y'_{AB}(x=2) = y'_{BC}(x=0)$$

$$\Leftrightarrow \frac{1}{EI}(120 \times 2) = \frac{1}{EI}\left(7.5\frac{0^3}{3} - 60\frac{0^2}{2} + 120 \times 0 + C_3\right) \Leftrightarrow C_3 = 240$$

→ Despreza-se a deformabilidade axial das barras $\Rightarrow \delta_B = 0$

$$\delta_B = 0 \Leftrightarrow y_{BC}(x=0) = 0 \Leftrightarrow C_4 = 0$$

1.7 → Lei das Rotações e Deslocamentos

$$y'_{BC} = \frac{1}{EI}\left(7{,}5\frac{x^3}{3} + 60\frac{x^2}{2} + 120x + 240\right)$$

$$y_{BC} = \frac{1}{EI}\left(7{,}5\frac{x^4}{12} - 60\frac{x^3}{6} + 120\frac{x^2}{2} + 240x\right)$$

1.8 → Valor de δ_1

$$\delta_1 = y_{BC}(x=4) = \frac{1}{EI}\left(7{,}5\frac{4^4}{12} - 60 \times \frac{4^3}{6} + 120 \times \frac{4^2}{2} + 240 \times 4\right) = \frac{1440}{EI} \text{ [m]}$$

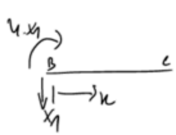
2 → Cálculo do valor de δ_2

Reações

$$\begin{cases} \Sigma F_x = 0 \Leftrightarrow H_A = 0 \\ \Sigma F_y = 0 \Leftrightarrow V_A + X_1 = 0 \Leftrightarrow V_A = -X_1 \ (\downarrow) \\ \Sigma M_A = 0 \Leftrightarrow -X_1 \times 4 + M_A = 0 \Leftrightarrow M_A = 4 \cdot X_1 \ (\curvearrowright) \end{cases}$$

Barra $\overline{AB}$ $\quad 0 \leq x \leq 2$

$$M_{AB}(x) = 4 \cdot X_1 \text{ kN/m}$$



Barra $\overline{BC}$ $= \quad 0 \leq x \leq 4$

$$M_{BC}(x) = 4 \cdot X_1 - X_1 \cdot x$$

2.1 → Integração da Linha Elástica ($\overline{AB}$) $y'' = -\dfrac{M}{EI}$

$$y''_{AB} = -\frac{M}{EI} = -\frac{1}{EI}\left(4 \cdot X_1\right) = \frac{1}{EI}\left(-4 \cdot X_1\right)$$

$$y'_{AB} = \frac{1}{EI}\left(-4 \cdot X_1 \cdot x + C_1\right)$$

$$y_{AB} = \frac{1}{EI}\left(-4 \cdot X_1 \cdot \frac{x^2}{2} + C_1 \cdot x + C_2\right)$$

2.2 → Condições de Fronteira ($\overline{AB}$)

→ Rotação nula $\Rightarrow \varphi_A = 0 \Rightarrow y'_{AB}(x=0) = 0 \Leftrightarrow C_1 = 0$

→ Deslocamento nulo $\Rightarrow \delta_A^H = 0 \Rightarrow y_{AB}(x=0) = 0 \Rightarrow C_2 = 0$

2.3 → Lei da Rotações e dos deslocamentos ($\overline{AB}$)

$$y'_{AB} = \frac{1}{EI}\left(-4 \cdot X_1 \cdot x\right) \quad ; \quad y_{AB} = \frac{1}{EI}\left(-4 \cdot X_1 \cdot \frac{x^2}{2}\right)$$

2.4 → Integração da Linha Elástica ($\overline{BC}$) $y'' = -\dfrac{M}{EI}$

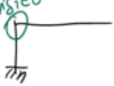
$$y''_{BC} = -\frac{M}{EI} = \frac{1}{EI}\left(X_1 \cdot x - 4 \cdot X_1\right)$$

$$y'_{BC} = \frac{1}{EI}\left(X_1 \cdot \frac{x^2}{2} - 4 \cdot X_1 \cdot x + C_3\right)$$

$$y_{BC} = \frac{1}{EI}\left(X_1 \cdot \frac{x^3}{6} - 4 \cdot X_1 \cdot \frac{x^2}{2} + C_3 \cdot x + C_4\right)$$

2.5 → Condições de Fronteira ($\overline{BC}$)

Nó Rígido



$$\varphi_B^{esq} = \varphi_B^{Dir} \Leftrightarrow y'_{AB}(x=2) = y'_{BC}(x=0) \Leftrightarrow \frac{1}{EI}\left(-4 \cdot X_1 \cdot 2\right) = \frac{1}{EI}\left(X_1 \cdot \frac{0^2}{2} - 4 \cdot X_1 \cdot 0 + C_3\right)$$

$$\Leftrightarrow C_3 = -8 \cdot X_1$$

⇒ Desprezando deformabilidade axial das barras $\Rightarrow \delta_B = 0 \Leftrightarrow y_{BC}(x=0) = 0 \Leftrightarrow C_4 = 0$

2.6 → Lei das Rot. e Desl. ($\overline{BC}$)

$$y'_{BC} = \frac{1}{EI}\left(X_1 \cdot \frac{x^2}{2} - 4 \cdot X_1 \cdot x - 8 \cdot X_1\right) \quad ; \quad y_{BC} = \frac{1}{EI}\left(X_1 \cdot \frac{x^3}{6} - 4 \cdot X_1 \cdot \frac{x^2}{2} - 8 \cdot X_1 \cdot x\right)$$

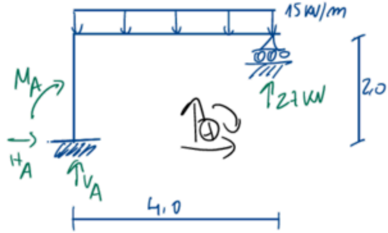
2.7 → Valor de δ_2

$$\delta_2 = y_{BC}(x=4) = \frac{1}{EI}\left(X_1 \cdot \frac{4^3}{6} - 4 \cdot X_1 \cdot \frac{4^2}{2} - 8 \cdot X_1 \cdot 4\right) = -\frac{160 \cdot X_1}{3\,EI}$$

3 → Valor da Incógnita Hiperestática (X_1)

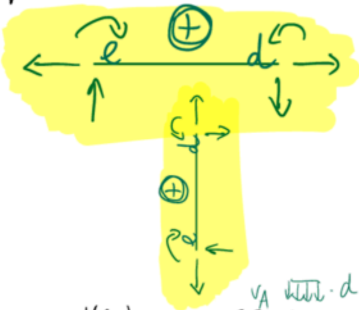
$$\delta_1 + \delta_2 = 0 \Rightarrow \frac{1440}{EI} - \frac{160 \cdot X_1}{3\,EI} = 0 \Leftrightarrow X_1 = 27\ \text{kN}$$

c) Calcular Reações e Traçar Diagramas de Esforços



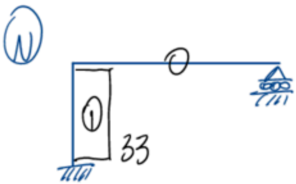
$$\begin{cases} \Sigma F_x = 0 \Leftrightarrow H_A = 0 \\ \Sigma F_y = 0 \Leftrightarrow V_A + 27 - 15\times 4 = 0 \Leftrightarrow V_A = 33\ kN\ (\uparrow) \\ \Sigma M_A = 0 \Leftrightarrow M_A + 15\times 4\times \frac{4}{2} - 27\times 4 = 0 \Leftrightarrow M_A = -12\ kN/m \end{cases}$$

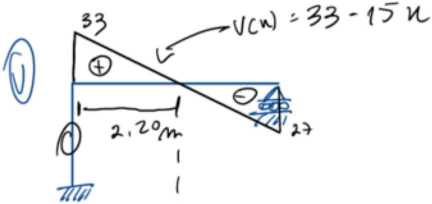
$(\circlearrowleft)$

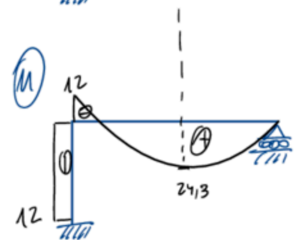


V_A ⊥⊥⊥ · d

$V(x) = 0 \Leftrightarrow 33 - 15x = 0 \Leftrightarrow x = 2,20\ m$





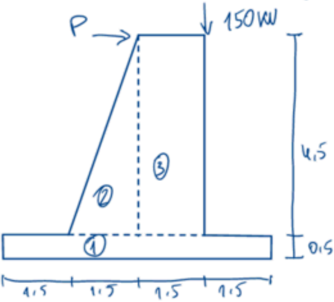


$M_A + V_A \times d - 15 \cdot x \cdot \frac{x}{2}$

$M_{máx}(x=2,20) = 12 + 33\times 2,20 - 7,5\times 2,2^2$

$= 24,3\ kN/m$

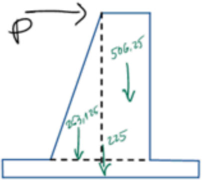
Exame 16/02/2024 ⇝ Problema 4

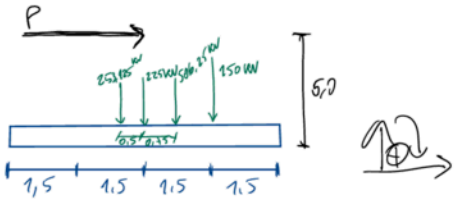


a) Redução das forças ao centro de gravidade da sapata:

1→) Resultante: R = 150 (Força Aplicada)
+
6×0,5×3,0×25 (P.P. da sapata ①)
+
$\frac{4,5\times1,5}{2}$ ×3,0×25 (P.P. do triângulo ②)
+
1,5×4,5×3,0×25 (P.P. do Retângulo ③)

(=) R = 150 + 225 + 253,125 + 506,25 = 1134,375 kN

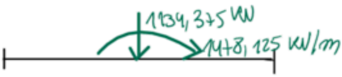




3→) Momento Resultante ⇒ Momento das forças em relação ao Centro de Gravidade da sapata

(=) M_R = 150 × 1,5 + 506,25 × 0,75 + 225 × 0 − 253,125 × 0,5 + P × 5,0

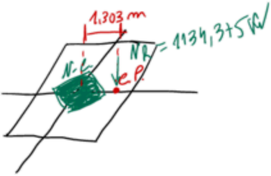
(=) M_R = 478,125 + 5×P → Quando P = 200 kN (=) M_R = 1478,25 kN/m (na base da sapata)

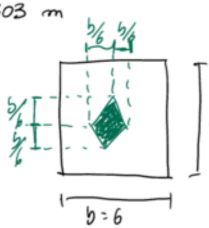


4→) Verificar se a sapata tem tensões de tração (eixo neutro cai dentro da secção)

· Se o C.P. estiver dentro do núcleo central as tensões têm todas o mesmo sinal.

C.P. = $\frac{M_R}{N_R}$ = $\frac{1478,125}{1134,375}$ = 1,303 m



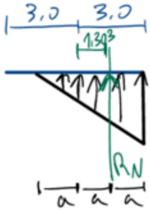


$\frac{b}{6} = \frac{6}{6} = 1,0$ m < 1,303 ✗

⇒ Centro de Pressões cai fora do Núcleo Central

Há Trações → Redistribuição de esforços

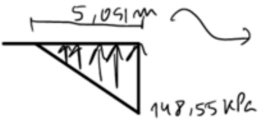
5 → Redistribuição dos esforços:



$$R_N = \text{A. do } \Delta \text{ de tensões} \times \text{largura} = \frac{\sigma_{máx} \times (3 \cdot a)}{2} \times 3{,}0$$

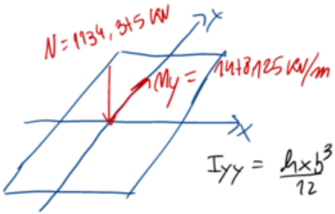
$$a = 3{,}0 - 1{,}303 = 1{,}697 \text{ m}$$

$$R_N = N_R \Leftrightarrow \frac{\sigma_{máx} \times (3 \times 1{,}697)}{2} \times 3{,}0 = 1134{,}375 \Leftrightarrow \sigma_{máx} = 148{,}55 \text{ kPa}$$

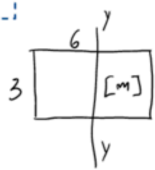


$\rightarrow 3 \times \underbrace{1{,}697}_{a}$ m

b) Posição do eixo neutro antes da redistribuição dos esforços:



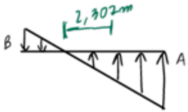
$$\boxed{\sigma = \frac{N}{A} \pm \frac{M_y}{I_y} \cdot x}$$



$$I_{yy} = \frac{h \times b^3}{12} = \frac{3 \times 6^3}{12} = 54 \text{ m}^4$$

Posição do eixo neutro: (Quando $\sigma = 0$) $\Leftrightarrow$

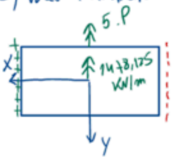
$$\Leftrightarrow -\frac{1134{,}325}{6 \times 3} \pm \frac{1478{,}125}{54} \cdot x = 0 \Leftrightarrow x = -2{,}302 \text{ m}$$

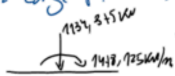


$$\sigma_A(x=3) = -\frac{1134{,}325}{6 \times 3} + \frac{1478{,}125}{54} \cdot 3 = 0 \Leftrightarrow \sigma_A(x=3) = -145{,}13 \text{ kPa}$$

$$\sigma_B(x=-3) = -\frac{1134{,}325}{6 \times 3} - \frac{1478{,}125}{54} \cdot (-3) = 0 \Leftrightarrow \sigma_B(x=-3) = 19{,}10 \text{ kPa}$$

c) Valor máximo da carga P. ⤳ Hipótese 1





$\sigma_A = 0 \Rightarrow \sigma(x=3) = 0$

? → usar regra polegar !!

$$\sigma_{(x=3)} = 0 \; \frac{1134{,}325}{18} \; ? \; \frac{1478{,}125 + 5 \times P}{54} \times 3{,}0 \Leftrightarrow P_{(máx)} = 131{,}25 \text{ kN} (\rightarrow)$$