Midterm

PSTAT 120B: Probability and Statistics
Winter 2025

Name
(First and Last)

NetID:

Section LI YUNING (W 8:00) PARK M (W 9:00) PARK M (W 10:00)
(Circle one)

PLEASE MAKE SURE TO FILL OUT THIS SECTION COMPLETELY.

INSTRUCTIONS:

- **Time Limit**: You have **60** minutes to complete this **4-**question assessment.

Please be sure to show all work; correct answers with no supporting work will not receive full credit.

- **Allowed Materials**: You may use **a calculator** (no internet connectivity), but no other materials (no laptops, phones, notes, etc.).
- You will be provided with a handful of notes including the main formulas, common probability distribution properties, and tables.
- **Information**: Remember to fill out your personal information above (e.g., name, NetID, section).

...::: GOOD LUCK :::...

Question	Points	Scores	Comments
1			
2			
3			
4			
Total			

ANSWER ALL THE FOLLOWING QUESTIONS:

Question 1:

a) Find the probability distribution of the sum of n independent random variables $X_1, X_2, \ldots, X_n$ having Poisson distributions with the respective parameters $\lambda_1, \lambda_2, \ldots, \lambda_n$.

b) Suppose $Z_i \sim \chi^2_{(2)}$ $(i = 1, \ldots, n)$ are independent random variables. What is the distribution of $U = \sum_{i=1}^{n} Z_i$? (State the distribution name and any relevant parameter(s)).

Question 2:

a) If two samples X_1 and X_2 with sizes n_1 and n_2 are generated from two normal populations $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$, respectively. Show that $\bar{X}_1 + \bar{X}_2 \sim N\left(\mu_1 + \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$.

b) b) A particular brand of drink has an average of 12 ounces per can. The amount of liquid in these bottles is normally distributed with $\sigma = 0.8$ ounce. How many bottles should be included in the sample if we wish $\bar{Y}$ to be within 0.5 ounce of μ with probability 0.99?

c) An economist wants to determine whether people are keeping their cars for longer now than they have in the past. She knows that, five years ago, about 38% of all passenger vehicles in operation were at least 10 years old. She commissions a study in which she randomly samples 325 passenger vehicles on the road and finds that, of the 325, 132 cars are 10 years or older.

Question 3:

a) Let Y has standard normal distribution, show that Y^2 has a chi-square distribution with degree of freedom, $\nu = 1$ (You may use the fact $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$).

b) The waiting time Y until delivery of a new component for an industrial operation is uniformly distributed over the interval from 1 to 5 days. The cost of this delay is given by $U = 2Y^2 + 3$. Find the probability density function for U using distribution function method.

Question 4:

a) Assume $Y_1, Y_2, \ldots, Y_n$ is a random sample from a continuous population with cdf $F(y)$ and pdf $f(y)$. Show that the first order statistic, or the sample minimum, $Y_{(1)}$ has the pdf: $f_{(1)}(y) = nf(y)[1 - F(y)]^{n-1}$.

b) Let $Y_1, \ldots, Y_n$ be a random sample of size n from a normal distribution with mean μ and variance σ^2. Show that $\frac{(n-1)S^2}{\sigma^2}$ has a chi-square distribution with $n - 1$ df.

...::: GOOD LUCK :::...